

## The Character of Scaling Earthquake Source Spectra for Kamchatka in the 3.5–6.5 Magnitude Range

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**Abstract**—The properties of the source spectra of local shallow-focus earthquakes on Kamchatka in the range of magnitudes  $M_w = 3.5–6.5$  are studied using 460 records of  $S$ -waves obtained at the PET station. The family of average source spectra is constructed; the spectra are used to study the relationship between  $M_w$  and the key quasi-dimensionless source parameters: stress drop  $\Delta\sigma$  and apparent stress  $\sigma_a$ . It is found that the parameter  $\Delta\sigma$  is almost stable, while  $\sigma_a$  grows steadily as the magnitude  $M_w$  increases, indicating that the similarity is violated. It is known that at sufficiently large  $M_w$  the similarity hypothesis is approximately valid: both parameters  $\Delta\sigma$  and  $\sigma_a$  do not show any noticeable magnitude dependence. It has been established that  $M_w \approx 5.7$  is the threshold value of the magnitude when the change in regimes described occurs for the conditions on Kamchatka.

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The source time function is a key to our ideas on the origin of earthquake sources. Usually it is seismic moment rate  $\dot{M}_0(t)$ ; its amplitude spectrum  $\dot{M}_0(f)$  is called the earthquake source spectrum (ESS). The important source parameters are the seismic moment  $M_0 = M_0(t)|_{t \rightarrow \infty} = \dot{M}_0(f)|_{f=0}$  and the  $M_0$ -related moment magnitude  $M_w = 2/3(\log M_0[\text{N m}] - 9.05)$ . Ideally, the functions  $\dot{M}_0(t)$  and  $\dot{M}_0(f)$  are directly related to the displacement in the  $P$ - or  $S$ -wave and its spectrum. The scaling properties of the ESS sets are interesting, in particular, the average ESS dependence on  $M_0$  or  $M_w$  (a scaling law). The idealized ESS scaling law can be considered in three equivalent variants, such as  $\dot{M}_0(f)$ ,  $\ddot{M}_0(f) = 2\pi f \dot{M}_0(f)$  and  $\ddot{\dot{M}}_0(f) = (2\pi f)^2 \dot{M}_0(f)$ . An important element of this law is the relationship between the characteristic (corner) ESS frequency  $f_c$  and  $M_0$ . For large values of  $M_w$  ( $M_w = 6–9$ ), usually  $f_c \sim M_0^{-1/3}$ . This type of scaling corresponds to the assumption of the geometrical and kinematic similarity of the differently-sized sources

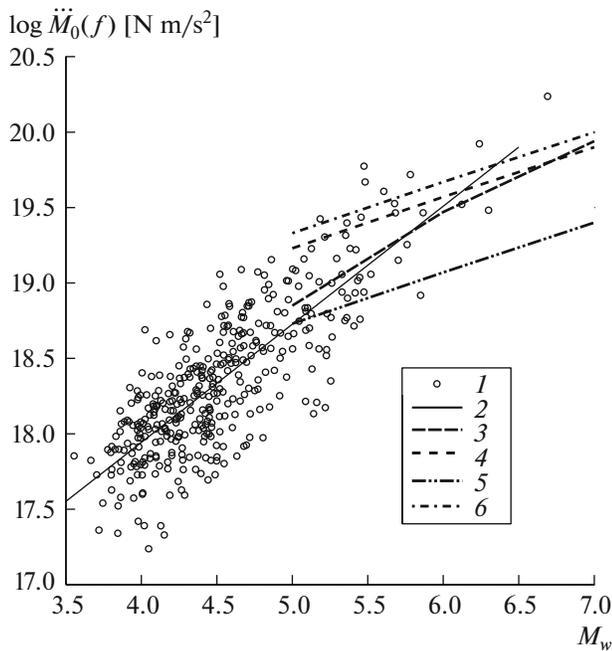
[1, 2]. In this case,  $M_0 \propto E_s \propto L^3 \propto B^3 \propto T^3 \propto f_c^{-3}$ , where  $E_s$  is the seismic energy,  $L$  is the source size,  $B$  is the average slip, and  $T$  is the duration. Here, the key quasi-dimensionless source parameters, such as stress drop  $\Delta\sigma \approx 2\mu B/L$  ( $\mu$  is the shear modulus) and apparent stress  $\sigma_a = \mu E_s/M_0$ , do not exhibit a systematic dependence on  $M_0$ . At smaller values of  $M_w$ , the situation is different. Several researchers ([3] etc.) believe that  $\Delta\sigma$  and  $\sigma_a$  are also stable here; therefore, the similarity exists. Another group [4] holds the opinion that the parameters  $\Delta\sigma$  and  $\Delta\sigma_a$  decrease together with  $M_0$ , and the similarity is violated. The discussion has already been continued for several decades, but the question has not been solved, though it is important for the physics of source processes and for the applications.

This question was studied based on the material of the  $S$ -wave spectra of the earthquakes on Kamchatka for a source using a specially determined model of  $S$ -wave attenuation. We constructed a family of average ESS for  $M_w = 3.5–6.5$  and studied the relationship between  $M_w$  and the parameters  $\Delta\sigma$  and  $\sigma_a$ . It was found that parameter  $\Delta\sigma$  is almost stable, while  $\sigma_a$  grows steadily, as the magnitude  $M_w$  increases. Thus, we identified the qualitative differences in the behavior of these parameters, and the question on the reality of the similarity of the sources on Kamchatka does not have a simple answer. When the values of  $M_w$  are quite large, the difference found vanishes: at  $M_w > 5.7$ , both

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**Fig. 1.**  $M_w$  dependences of the acceleration  $\log \ddot{M}_0(f)_{\max}$  spectral level. 1, data; 2, linear approximation of  $\ddot{M}_0(f)_{\max} \propto M_0^{0.52} \propto 10^{0.38M_w}$  type; 3, trend for the range of  $M_w = 5-7$  for Kamchatka based on [9]; 4-6, similar trends for (4) mantle sources near Hokkaido, (5) crustal sources near Honshu, and (6) mantle sources near Honshu.

parameters  $\Delta\sigma$  and  $\sigma_a$  show no noticeable dependence on magnitude.

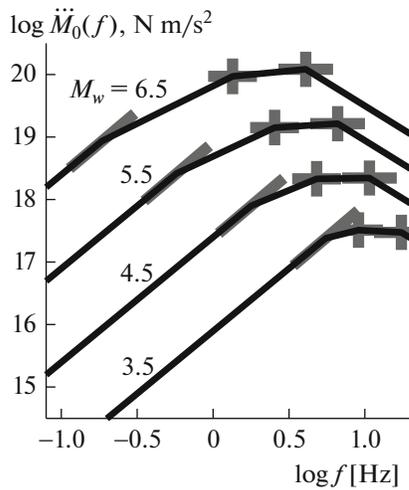
In this work, we used 460 records of Kamchatka earthquakes with the magnitudes  $M_L = 4-6.5$  ( $M_w = 3.5-6.5$ ) and the source depths of 0-70 km at the hypocentral distances  $r = 80-220$  km. Acceleration with a sampling frequency of 80 or 100 Hz was recorded in 1993-2012 at the Petropavlovsk seismic station (PET). The amplitude  $S$ -wave spectra were calculated by a window function, then they were averaged with respect to the NS, EW channels and the points of the discrete spectra within the 1/3 octave band width. The smoothed spectra were reduced to  $r = 1$  km by compensation of the geometric spreading and the losses on the ray trace. If we accept the attenuation parameters according to [5] (the spreading of  $1/r$  kind), the parameters of losses are the following:  $\kappa_0 = 0.034$  s and the quality factor  $Q(f, r) = 164f^{0.59}(1 - 0.0017(r - 100))^{-1}$ . The obtained reduced acceleration spectra  $A(f)$  were transformed to the velocity  $V(f)$  and displacement  $\Omega(f)$  spectra. The spectrum  $\Omega(f)$  was used to calculate the ESS as  $\dot{M}_0(f) = C_1\Omega(f)$  and similarly  $V(f)$  and  $A(f)$ , to find  $\dot{M}_0(f)$  and  $\ddot{M}_0(f)$ ; here, the constant  $C_1 = 3.24 \times 10^{18}$ . The numerical values of  $\log \dot{M}_0(f)|_{f=0} = \log M_0$

are close to  $\log M_0$  (GCMT); on average,  $M_w - M_w(\text{GCMT}) = -0.17$ . A piecewise linear approximation (in a bilogarithmic scale) was selected for each spectral curve obtained. For the individual spectra, we estimated the levels of spectral maxima  $\log \dot{M}_0(f)|_{f=0}$ ,  $\log \ddot{M}_0(f)_{\max}$ , and  $\log \dot{M}_0(f)_{\max}$ , as well as the corner frequencies  $f_{c1}$ ,  $f_{c2}$ , and  $f_{c3}$ . Examples of the processing procedure are shown in [6].

Similarly to the standard  $\omega^{-2}$ -models of  $\dot{M}_0(f)$  according to [1, 2], the accepted model includes a flat segment ( $\propto f^0$ ) at low frequencies (below  $f_{c1}$ ) and a segment with the  $f^{-2}$  behavior at high frequencies. However, unlike the  $\omega^{-2}$ -model, the segments with the  $\propto f^0$  and  $\propto f^{-2}$  behavior are not mated directly at  $f = f_c$ : there is an intermediate segment of the  $f^{-(1-1.5)}$  type that is bounded by the bends on both sides at  $f_{c1}$  and  $f_{c2}$  [2]. A typical segment of the  $f^{-2}$  type between  $f_{c2}$  and  $f_{c3}$  is found above  $f_{c2}$ . Above  $f_{c3}$ ,  $\dot{M}_0(f)$  falls off according to  $f^{-(3-4)}$ . In this case, each frequency  $f_{c1}$ ,  $f_{c2}$ , and  $f_{c3}$  follows its own trend. This model of scaling was proposed at the conceptual level in [7]; the significant observational material that maintains the idea of scaling with three corner-frequencies was generalized in [8]. The data for the Kamchatka spectra are presented in [6, 9]. An important feature for the scaling variant from Fig. 1 is the magnitude dependence of the scaling character for the level of  $\log \ddot{M}_0(f)$ . For this level, the rate of growth with  $M_w$  is switched from fast at  $M_w = 3-5.5$  to a slower one at  $M_w = 5.5-7.5$ . However, for the important parameter of the source  $f_{c1}$ , it is assumed that the relationship  $f_{c1} \propto M_0^{-1/3} \propto 10^{-0.5M_w}$  is fulfilled at any  $M_w$ .

In order to check the validity of the accepted concept of the spectral scaling, we studied the empirical  $M_w$  dependence for its parameters. For the frequencies  $f_{c1}$ ,  $f_{c2}$ , and  $f_{c3}$ , this dependence was studied in [6, 10]. The dependence for the levels of  $\log \ddot{M}_0(f)_{\max}$  is shown in Fig. 1. These dependences were approximated by the linear functions. For  $f_{c1}$ , the regression showed the law  $f_{c1} \propto M_0^{0.33 \pm 0.02}$ ; in this case, the value is close to 1/3, which is the value expected for the case of similarity. For  $f_{c2}$  and especially for  $f_{c3}$ , the slope of the trends is more gradual, which is not consistent with the idea of similarity.

The family of the average spectra was constructed using the dependences described (Fig. 2). Here, the key numerical parameter is the growth rate of the level of the area  $\log \ddot{M}_0(f)_{\max}$  with  $\log M_0$ , which is found from the slope of straight line 2 in Fig. 1. As a result,  $\ddot{M}_0(f)_{\max} \propto M_0^{0.52}$ ; this value exceeds the value of 1/3 expected according to the similarity hypothesis by 1.55.



**Fig. 2.** Family of average ESS  $\ddot{M}_0(f)$  for the Kamchatka earthquakes. Gray segments are the ranges of the root-mean-square scatter of individual spectra with respect to the level and position of the corner point.

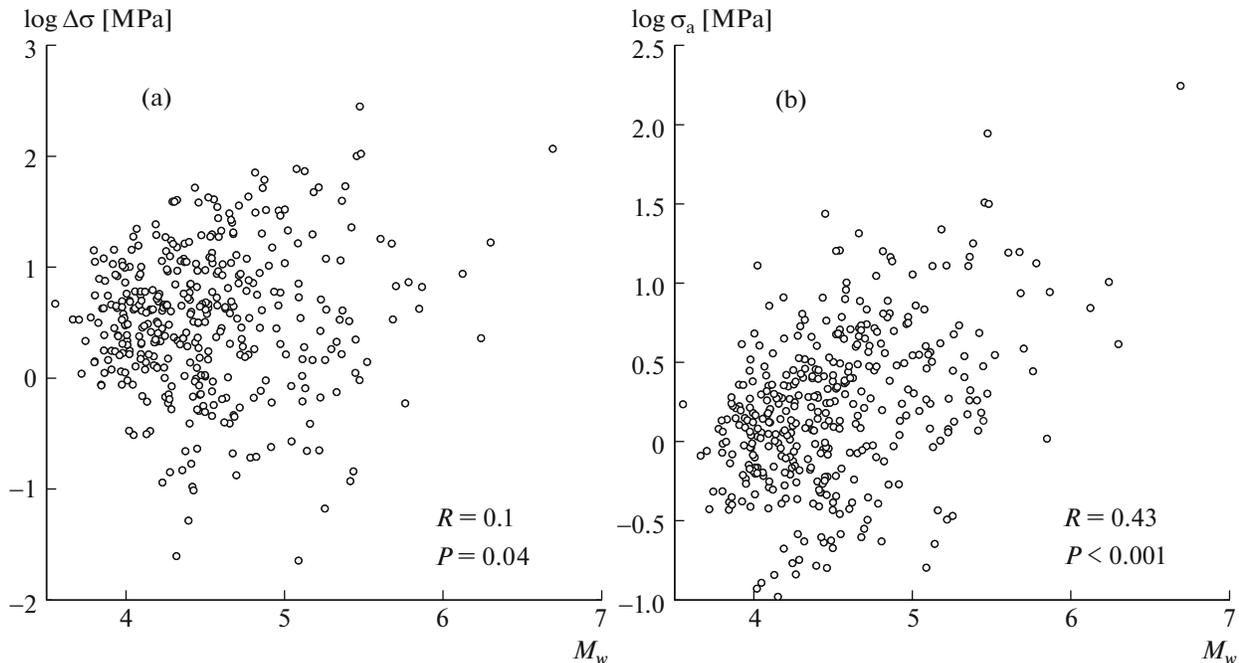
Figure 1 shows that the trend recorded for the 3.5–6 magnitude range obviously contradicts the trend for the range of  $M = 6–7$ , which can be estimated for the Kamchatka earthquakes using the average acceleration spectra from [9]. However, in the range of  $M = 5.5–6$ , these trends are matching. We may conclude that a “crossover” occurs at  $M_w \approx 5.7$ : the fast growth of the level of the high-frequency radiation is terminated, and “the regime of strong earthquakes” origi-

nates when the properties of similarity are fulfilled approximately.

In order to clarify the question about similarity, it is useful to study the behavior of  $\Delta\sigma$  and  $\sigma_a$ . The value of  $\Delta\sigma$  was estimated by using the spectral method (after Brune [2]) and the formula  $\Delta\sigma = 8.47M_0(f_{c1}/c_s)^3$  [11], where  $c_s$  is the velocity of the  $S$ -waves (it is accepted that  $c_s = 3.8$  km/s). The  $\Delta\sigma(M_w)$  dependence (Fig. 3a) is expressed weakly; even the correlation is unlikely to occur (the level of significance is 4%); therefore, even if the relationship exists, it is weak. We note that  $\Delta\sigma$  is usually calculated using the estimate of  $f_c$ , which is close to  $(f_{c1}, f_{c2})^{0.5}$  and does not have a clearcut meaning, rather than  $f_{c1}$  [12]. Under this approach, an approximate correlation between the behavior of the  $\Delta\sigma$  estimates and the similarity hypothesis does not take place.

In order to find  $\sigma_a$  determined as  $\mu E_s/M_0$ , we assume  $\mu = 70$  GPa and calculate the energy by the formula  $E_s(J) = 1.04 \times 10^{18} \Delta f V_{\max}^2$ , where  $\Delta f$  [Hz] =  $f_{c2} - f_{c1}$  and  $V_{\max}$  [m] are the width of the spectrum band  $V(f)$  and its peak amplitude, respectively. The  $\sigma_a(M_w)$  dependence is shown in Fig. 3b. It is seen that the correlation between  $\sigma_a$  and  $M_w$  is quite clearly manifested, the corresponding level of significance is less than 0.1%, and its reality is unquestionable.

These facts indicate a new vision of an old debate. At the very least, in several cases, the question on the validity of the hypothesis of similarity between sources for weak ( $M_w < 5.5–6$ ) earthquakes may have a simple



**Fig. 3.** The  $M_w$  dependence for (a)  $\Delta\sigma$  and (b)  $\sigma_a$ .

and unexpected answer: the deviations from this hypothesis are small for the “stress drop” parameter and at the same time are clearly manifested for the “apparent stress” parameter.

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