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==================================GEOPHYSICS==

Scaling properties of corner frequencies of Kamchatka earthquakes

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ABSTRACT

Scaling properties of earthquake populations bear the major information on the physics of the source process of an earthquake. To determine scaling properties, source spectra of more than 400 earthquakes of Kamchatka were determined in a frequency range 0.1-30 Hz using materials of digital registration of PET station, and characteristic frequencies of earthquakes were estimated. The range of magnitudes is 4-6.5, the range of distances is 80-220 km. To enable reduction of a spectrum to the source, attenuation properties of the medium around PET were determined beforehand. It is revealed that source spectra show several corner (characteristic) frequencies: f_{c1}, f_{c2} and f_{c3} ; where the spectral trend changes: from f^0 to f^{-1}, from f^{-1} to f^{-2}, and from f^{-2} to f^{-3}, respectively. Although in some cases $f_{c1} \approx f_{c2}$ in agreement with the usual ω^{-2} spectral model, the main part of spectra has more complicated character. For a large part of the studied earthquakes a source-controlled upper cutoff of acceleration spectrum, or corner frequency f_{c3} , is observed. This is an important fact, as the existence of f_{c3} (source-controlled $f_{\max}$) is not recognized in the bulk of the seismological literature. For f_{c1} , the observed scaling agrees with the usual hypothesis of similarity of the earthquake sources of different size (magnitude); it is close to $f_{c1} \propto M_0^{-1/3}$., where M_0 is seismic moment. For f_{c2} , scaling is close to $f_{c2} \propto M_0^{-0.17} \propto f_{c1}^{0.5}$, that indicates an expressed violation of similarity. For f_{c3}, scaling is close to $f_{c2} \propto M_0^{-0.08} \propto f_{c1}^{0.25}$, so that similarity is broken even sharper in this case. Hypotheses about possible causes of the observed scaling are discussed.

Keywords: earthquake source, earthquake source spectrum, scaling, violation of scaling, corner frequency, similarity, f-max

Seismograms represent the main source of information on the earthquake source process. Body wave displacement waveform reflects source time history, but with certain amplitude and phase distortions. Amplitude distortions related to loss along propagation path are corrected relatively easily, but phase distortions are difficult to compensate. Therefore it is convenient to study, instead of source time function, its amplitude spectrum called "earthquake source spectrum" (ESS). Seismological studies have shown that empirical ESS do not show expressed peaks, thus amplitude spectra can be safely smoothed, resulting in stabilized estimates. The standard model for ESS [1,2] is the ω^{-2} model that includes a flat part at low frequencies $f\,(\propto f^0)$ and a decay, of f^{-2} kind, at high frequencies (HF). These ranges are separated by a knee characterized by corner frequency f_{c1} (Fig. 1).

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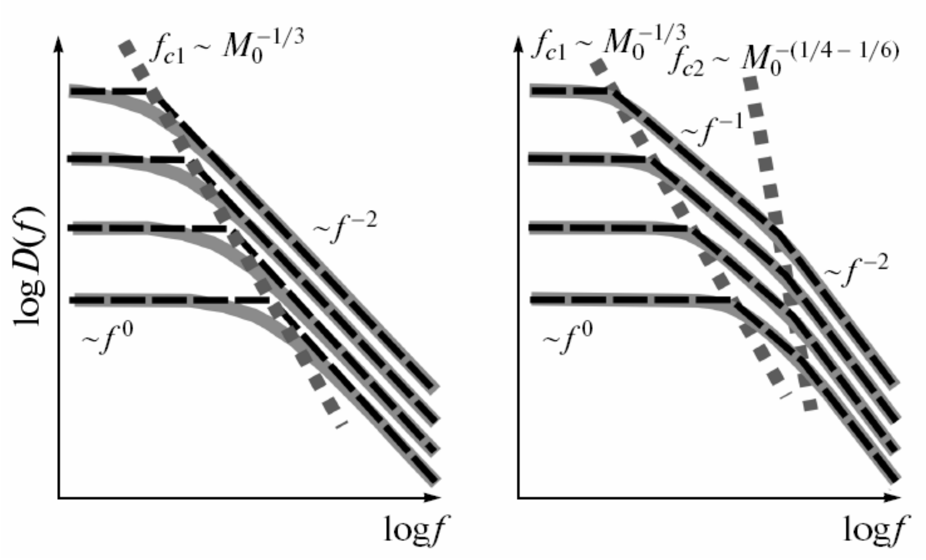

Fig.1. Sketch of scaling of source spectra according to the standard ω^{-2} model (left box), and according to the proposed concept (right box).

Of great interest is the study of scaling (the generalized similarity) of earthquake source spectra as it bears key information on properties of earthquake sources. Scaling for f_{c1} it is well explored and often close to $f_{c1} \propto M_0^{-1/3}$, where M_0 is the seismic moment of an earthquake. This kind of scaling corresponds to the assumption of geometrical and kinematic similarity of sources of different size; thus $M_0 \propto L^3 \propto T^3 \propto f_{c1}^{-3}$, where L is source size, and T is duration of the source process. Use of M_0 as the main scale parameter is related to the fact that that M_0 and related moment magnitude $M_w = 2/3 \lg M_0[\text{dyne}\cdot\text{cm}] - 10.7$ are basic source parameters in seismology, rather reliably determined.

It is noted in [2], that the mentioned knee of a spectrum can split itself into two, with corner frequencies f_{c1} and f_{c2}; in the range between these ESS has intermediate slope, close to f^{-1} (see Fig.1). In [3] the empirical scaling model for ESS is constructed, and it was found that for the majority earthquakes such intermediate range exists, and that scaling for f_{c2} is qualitatively different from that for f_{c1}. According to [4], based on published data of limited accuracy, $f_{c2} \propto f_{c1}^{0.5-0.7}$.

From source time function one can pass to its second derivative and to its spectrum – earthquake acceleration source spectrum (EASS). In the ω^{-2} model, EASS grows as f^2 below f_{c1} and shows a plateau above it. In observed acceleration spectra, the upper cutoff (f_{max}) is systematically found; usually it is attributed to frequency-dependent loss along wave propagation path. But in some cases such cutoff is manifested also in spectra corrected for propagation loss. In [3,5] the assumption is made, that this cutoff is defined by the source; such cutoff is further denoted f_{c3}. At f_{c3}, the HF decay of ESS is switched from the trend f^{-2} to approximately f^{-3}, and EASS shows a crossover switching from f^0 to f^{-1}. The existence of f_{c2} is partly acknowledged in modern seismological literature, but frequently is ignored. The existence of f_{c3} is recognized rarely. There are some limited studies elucidating scaling behavior of f_{c2}, whereas with respect to f_{c3} even reality of scaling is far from being established. In the present study an attempt is undertaken to clarify these questions, studying *S*-wave spectra of local earthquakes of Kamchatka. The basic results of the research are as follows:

(1) Several characteristic (corner) frequencies: f_{c1}, f_{c2} and f_{c3} are revealed in source spectra, where the spectral trend changes: from f^0 к f^{-1}, от f^{-1} к f^{-2}, and from f^{-2} to f^{-3}, respectively. Although in some cases $f_{c1} \approx f_{c2}$, in agreement with the common ω^{-2} spectral model, the majority of spectra has more complicated character.

(2) For the most part of the studied earthquakes an upper cutoff of source acceleration spectrum, i.e. f_{c3} corner frequency, is observed. It is the important fact because mere existence f_{c3} is not admitted in the bulk of the seismological literature.

(3) For f_{c1} the observed scaling is in accordance with the usual hypothesis of similarity, being close to $f_{c1} \propto M_0^{-1/3}$. For f_{c2} , scaling is established and occurred to be close to $f_{c2} \propto M_0^{-0.17} \propto f_{c1}^{0.5}$, that demonstrates expressed violation of similarity. For f_{c3} scaling also is established, being close to $f_{c3} \propto M_0^{-0.08} \propto f_{c1}^{0.25}$, thus similarity is broken in this case even more sharply.

To determine spectral shape and further to estimate characteristic frequencies of a source spectrum, the critical step is to accurately compensate frequency-dependent attenuation along signal propagation path. In [6] for the medium around seismic station PET the following estimates of parameters of S-wave attenuation were obtained: κ_0=0.016, $Q(f) = Q_0 f^{0.42}$ above 1 Hz; $Q(f) = Q_0$ below 1 Hz. The quality factor Q_0 was found to depend significantly on hypocentral distance r; the value of Q_0 used in calculations follows empirical formula $Q_0(r) = 165(0.52 + 0.48r/100)$.

For the study, more than 400 records of earthquakes of Kamchatka with magnitudes M_L = 4 - 6.5 ($K^{\Phi 68}$ = 9-13), were used, with r = 80-220 km. Records with sampling frequency 80 cps are recorded in 1993-2005 by seismic station "Petropavlovsk" (PET) with the digital channel: accelerometer FBA-23 – data logger "Quanterra". Amplitude spectra of records in the selected S wave window were calculated with the use of 10% taper, then averaged over two horizontal channels and over spectral points within each of the frequency bands that jointly covered the spectral range; the bands had fixed log-width equal to 0.1 decade (1/3 octave) (Fig. 2). From thus obtained acceleration spectra $A(f)$ (reflect EASS), velocity spectra $V(f)$ and displacement spectra $D(f)$ (directly reflect ESS) were determined. The spectral curves were approximated by piecewise linear trends (in log-log scale). Near-optimal fit of spectral shapes was obtained manually via graphical interface, and allows to estimate corner frequencies of a spectrum. The working range of frequencies is limited: from the low frequency side, at 0.5-0.2 Hz, by the energy burst related to unaccounted for contribution of surface waves; from the HF side - by the zone of inadmissible low signal/noise ratio. (Fig. 2).

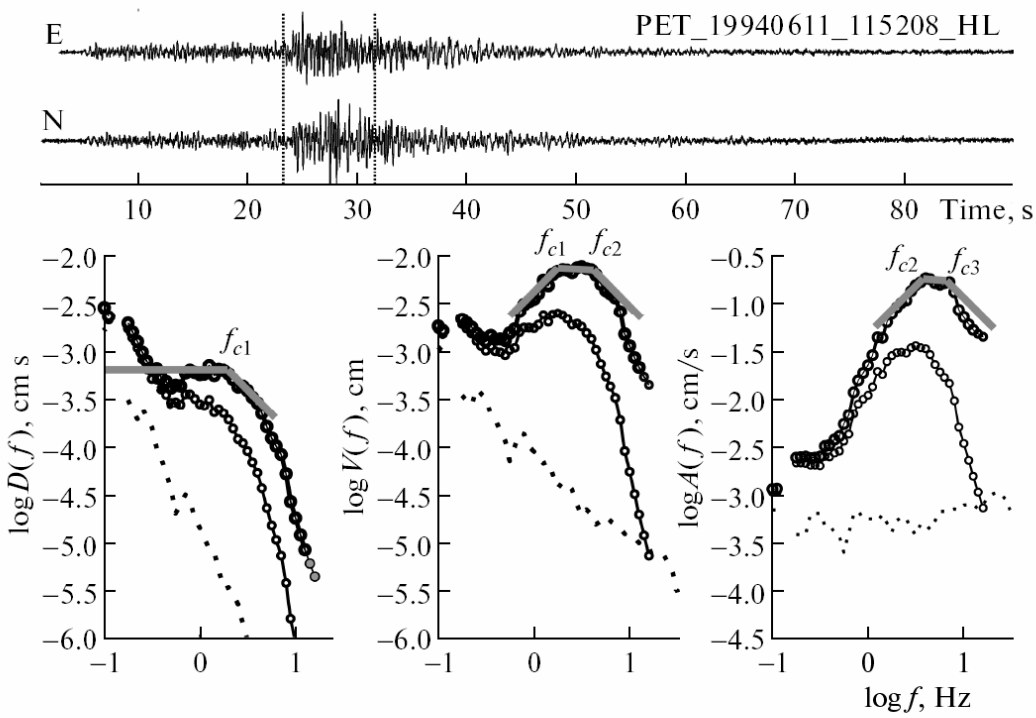

Fig. 2. An example of data processing. Above–initial acceleration records, horizontal components (amplitude in arbitrary units);the selected S_wave data window is marked. Below: spectra $D(f)$, $V(f)$ and $A(f)$, reduced to r = 1 km. Spectra are given in two vari_ants: raw (the lower curve) and corrected for attenuation (the upper curve). In the lower part of boxes—corresponding noise spectra (dotted line). The grey line approximates the observed spectrum.

To verify the quality of attenuation correction one can use the fact that in a considerable part of cases the f_{c3} corner-frequency cannot be observed even at sufficiently high level of the signal. This may mean either that f_{c3} it is localized above the upper observable frequency of the channel (30 Hz), or lost in noise, or, maybe, does not exist at all. In any of these cases, one can believe that the standard ω^{-2} spectral asymptotics is valid: ESS $\propto f^{-2}$; EASS is constant (flat). The fact, that after attenuation correction one can quite often observe at HF flat A (f) spectra (see Fig. 3a for example) indicates that there are no gross errors in the compensation of attenuation. This means that the estimates of f_{c3}, in cases when this feature is observable at all, can be considered as reliable. Therefore one can pass to more meaningful analysis of data.

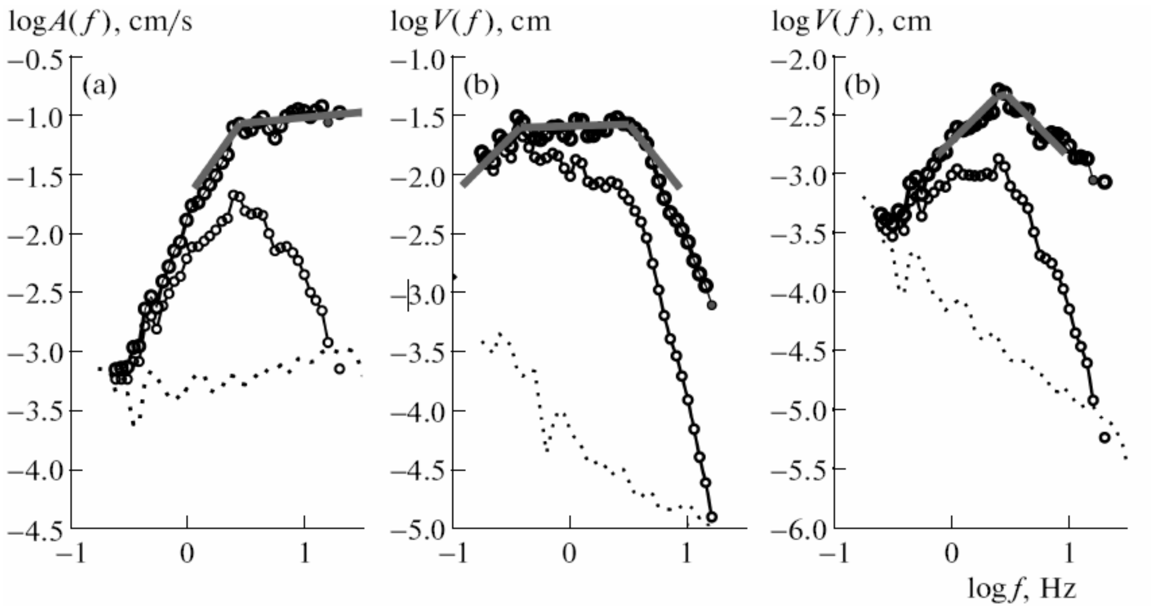

Fig. 3. Examples of spectra $A(f)$ (a) and $V(f)$ (b), (c), illustrating typical spectral shapes.

The first question is whether a separate f_{c2} corner is real. In such a case the $V(f)$ spectrum should have a flat part at its maximum. It was found that among the studied spectra there are many cases when there is an expressed broad flat maximum $V(f)$; in such cases the presence of two separate knees of a spectrum at $f = f_{c1}$ and at $f = f_{c2}$ is doubtless (see Fig 3b for example). But there are a lot of $V(f)$ spectra with a narrow peak, then f_{c1} and f_{c2} approximately coincide and the spectrum follows the ω^{-2} model (see Fig. 3c for example). There are also many intermediate cases; see example case on Fig 2.

The second question is whether f_{c3} exists. In such cases spectra A (f) should have the expressed upper cutoff. In many cases a cutoff of this kind is indeed observed; see Fig 2 for an example. Taking into account certain scatter of Q values among paths from different hypocenters, estimates of f_{c3} exceeding 18 Hz, were considered as the insufficiently reliable. Such data were put into the category «f_{c3}> 18 Hz», and into the same category the cases were also included when f_{c3} is not seen at all, and also cases when the spectrum A (f) did show some bend downwards but the accurate location of the corner was impossible to fix.

The scaling behavior of the resulting estimates f_{c1}, f_{c2} and f_{c3}, expressed by the general relationship $f_{ci} \propto M_0^{\beta_i}$, i =1,2,3, can be observed on Fig. 4 where the estimates f_{ci} are represented vs. local magnitude M_L, close to M_w. One can see, that the data on f_{c1}, Fig. 4a, show relatively wide scatter, and in general do not contradict the similarity hypothesis ($\beta_1 \approx 1/3$). The accurate estimation of the scaling exponent β_i is difficult because of the uncertainty in the exact value of the balance of scatter of the points on Fig 4a along abscissa and ordinate. For f_{c2} (Fig. 4b), again on the background of scatter, the slope of magnitude trend is clearly more gradual as compared to that for f_{c1}; $\beta_2 \approx 0.17$, that indicates to an approximate relationship $f_{c2} \propto f_{c1}^{0.5}$. Similarity is obviously broken.

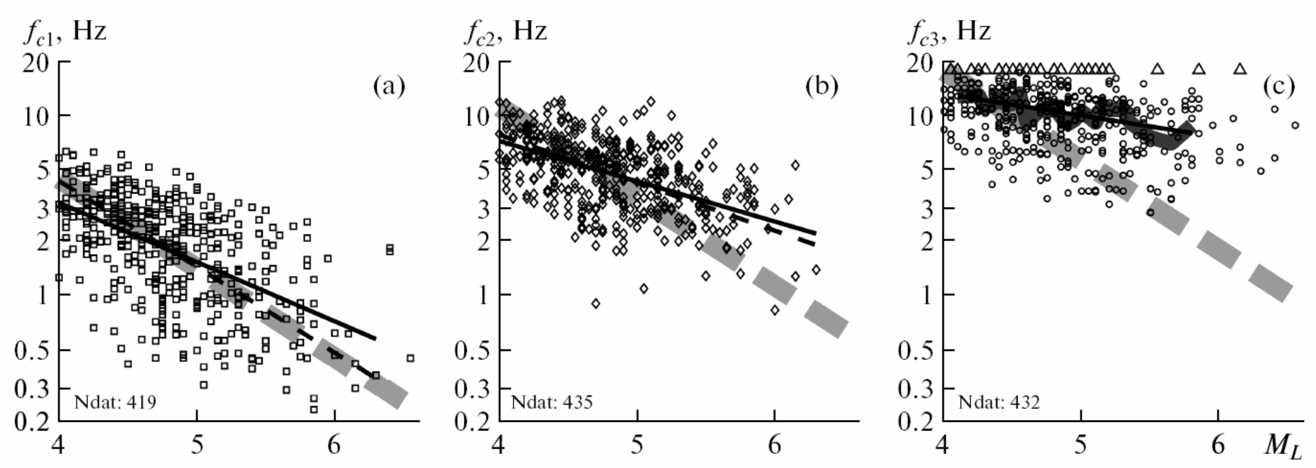

Fig. 4. The estimates of corner frequencies f_{c1}, (a) f_{c2} (b) and f_{c3} (c) vs. magnitude. Grey dashed lines correspond to the scaling $f_{ci} \propto M_0^{-1/3}$, expected in the case of similarity of earthquake sources. Continuous lines are the estimates of observed scaling obtained by means of common linear regression; the estimates of the scaling exponents are β_1=0.22±0.016 (a), β_2=0.15±0.011 (b) and β_3=0.081±0.013 (c). For a part of data on (c) only estimates from below (triangles) were available, therefore in order to estimate the trend, running median (the dark grey curve) was found beforehand, and its points were used to determine the linear trend. The obtained estimates β_i are somewhat biased down as compared to the hypothetic case when the values of M_L are exact; real accuracy of estimates β_i also is somewhat lower. Straight dashed lines show estimates of scaling trend calculated by orthogonal regression assuming $\sigma^2(M_L)$ =0.5 $\sigma^2(f_{ci})$; corresponding more realistic estimates of exponents are β_1=0.32 (a) and β_2=0.17 (b).

On Fig 4c one can see the relationship f_{c3} (M_L). The cases of the category «f_{c3}> 18 Hz» are marked by triangles put at f=18 Hz. Such «clipping» or «winsorization» does not prevent to notice the tendency to decay of f_{c3} with magnitude. To reveal it reliably, medians of the data f_{c3} (M_L) were calculated in moving window. Medians are not biased by clipping until it affects less than 50 % of data within a window, and clipping never was so strong. Linear regression applied to medians resulted in the estimate β_3=0.08; it approximately corresponds to the relationship $f_{c3} \propto f_{c1}^{0.25}$. Therefore, for f_{c3} similarity is broken even more sharply than for f_{c2}.

Thus, having analyzed hundreds of spectra it occurred possible to reveal the presence of little-studied characteristic frequencies f_{c2} and f_{c3}, and to establish the fact of the obvious difference of their scaling as compared to that of the common corner frequency f_{c1}; scaling exponents for f_{c2} and f_{c3} also differ significantly one from another. The deep causes of the observed differences of scaling from a behavior expected in the case of full similarity can be: for f_{c2} – broadening of the front of propagating rupture caused by a stochastic mechanism close to random walk [4]; for f_{c3} - presence of a low limit in the size distribution of inhomogeneities of the fault surface, with the position of this limit slowly increasing with magnitude. The estimate $\beta_1 \approx 1/3$ says that for sizes of the sources of the studied earthquake population there is no expressed violation of similarity; this fact indicates relative stability of the dimensionless source parameter «strain drop» and also presents certain interest.

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