Скейлинг и подобие
для параметров очагов землетрясений
и для очаговых спектров

Вводный обзор для аспирантов и молодых ученых

А.А.Гусев

Институт вулканологии и сейсмологии, Петропавловск-Камчатский.

Scaling and similarity for parameters of earthquake sources and of source spectra
(a review)

Alexander A. Gusev [1,2]

[1] *Institute of Volcanology and Seismology, Russ.Ac.Sci. Petropavlovsk-Kamchatsky,Russia*
[2] *Kamchatka Branch, Geophysical Service, Russ.Ac.Sci. Petropavlovsk-Kamchatsky,Russia*

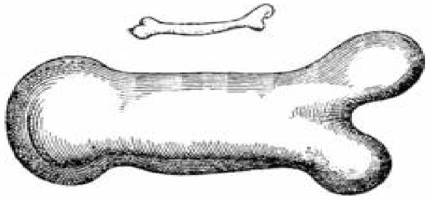
Moscow 2015

Principles
for analysis of scaling

(1) Analysis of scaling is a powerful approach in physics, capable to clarify behavior of weakly accessible objects or ones with less clear physics or incomprehensible mathematics. (Examples: hydrodynamics, explosion, turbulence)

(2) One have to select key dimensional parameters, and study their interrelationships, often of power law kind

(3) A specific instrument is extraction of dimensionless parameters



q / Galileo's original drawing, showing how larger animals' bones must be greater in diameter compared to their lengths.

Подобие и скейлинг

до Галилея: **подобие:**

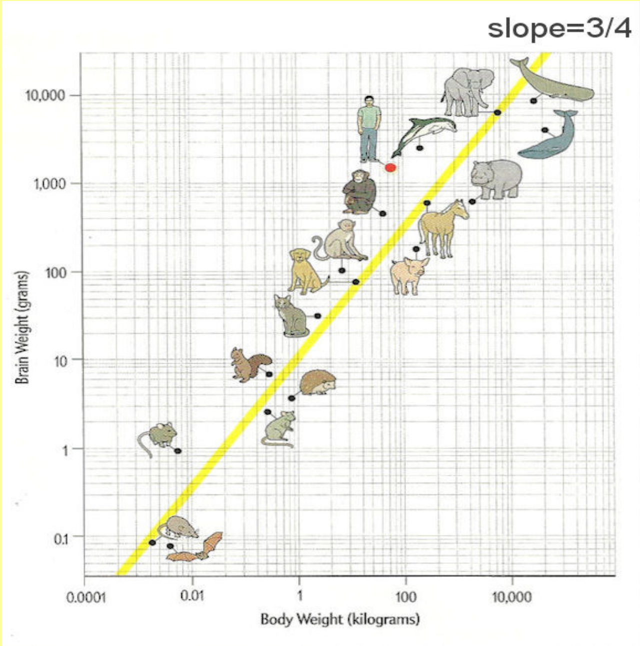
(масса скелета)~(масса тела)^{1}

????

после Галилея: **скейлинг есть;**

но подобие нарушено:

(масса скелета)~(масса тела)$^{1+x}$



скейлинг есть;

но подобие нарушено:

(масса мозга)~(масса тела)$^{1-x}$

before Galileo: **similarity**

(skeleton mass)$\propto$(body mass)^{1}

????

after Galileo: **scaling is present but similarity is absent:**

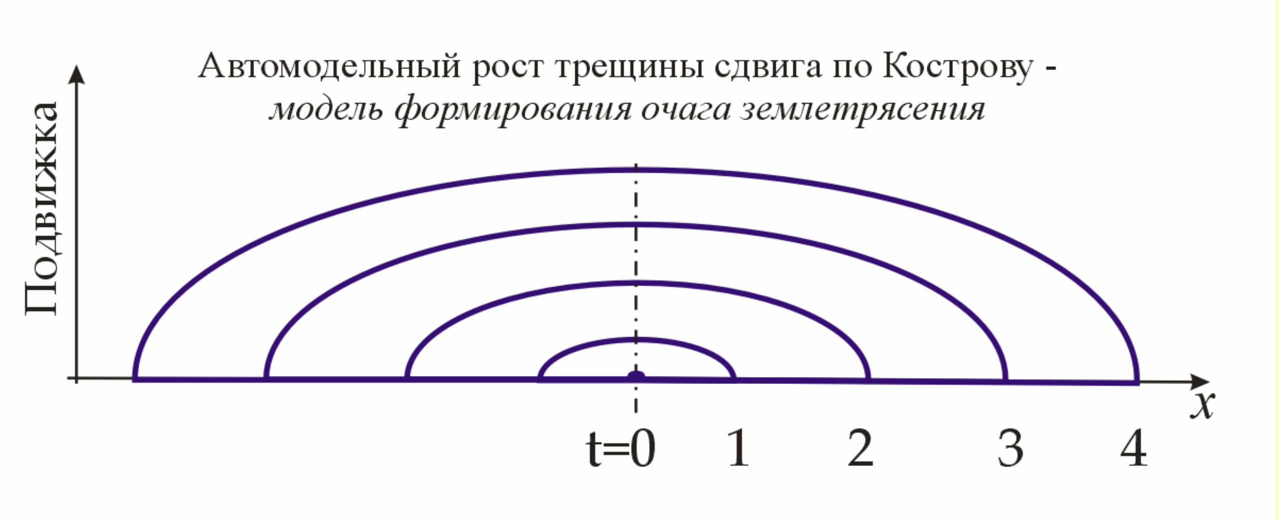
(skeleton mass)$\propto$(body mass)$^{1+x}$

scaling is present but similarity is absent:

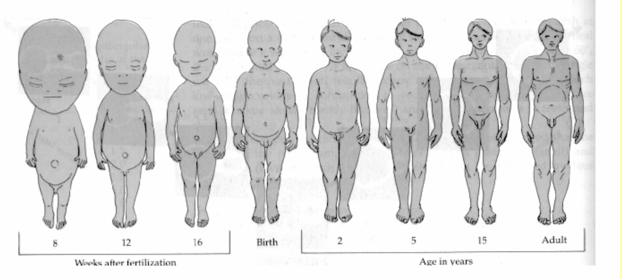
(brain mass)$\propto$(body mass)$^{1-x}$

Автомодельный рост – подобие стадий роста
нередко - удобное приближение
но *его применимость в реальной задаче может быть спорной;*

Self-similar growth – similarity og stages of growth; often – a good approximation
but its applicability need check in practical situation



Slip=f(distance) self-similar growth of shear crack after Kostrov as a model for formation of an earthquake source

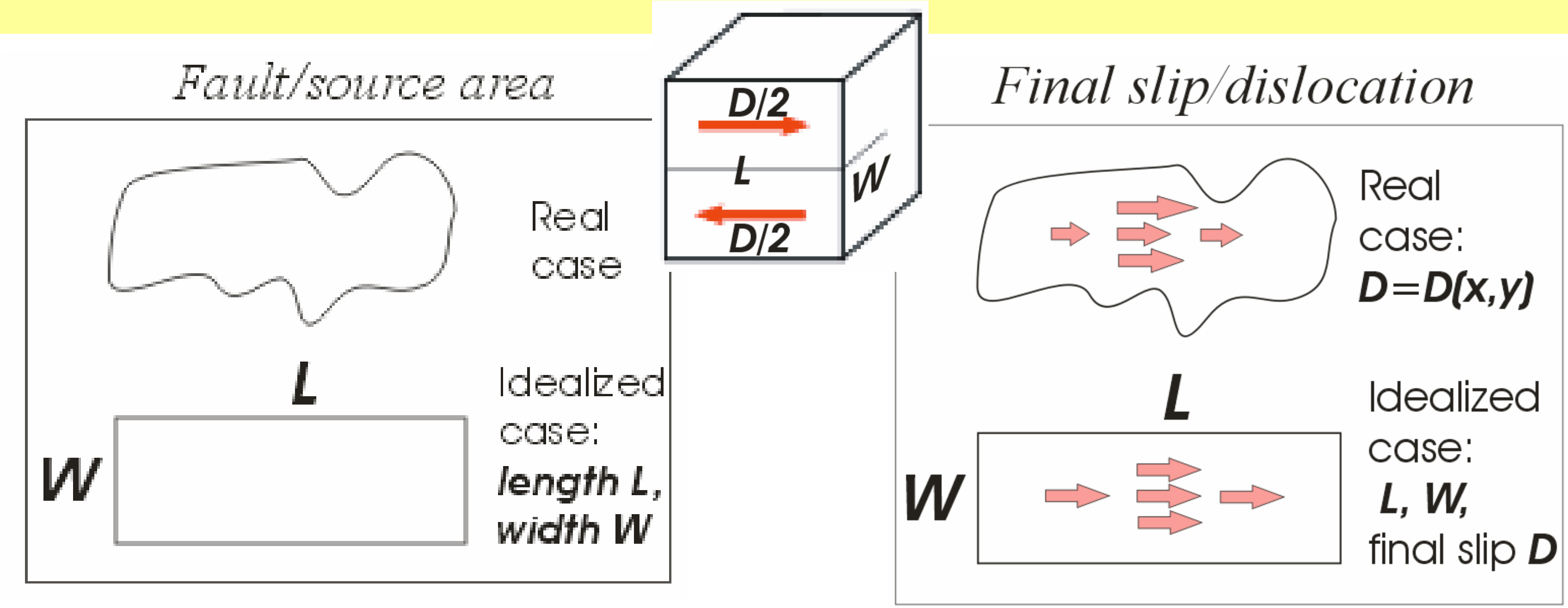


в сложной системе

автомодельности нет

no self-similarity in a complex system

Key dimensional parameters
A. Geometry



Area: $S=L\cdot W \propto L^2$

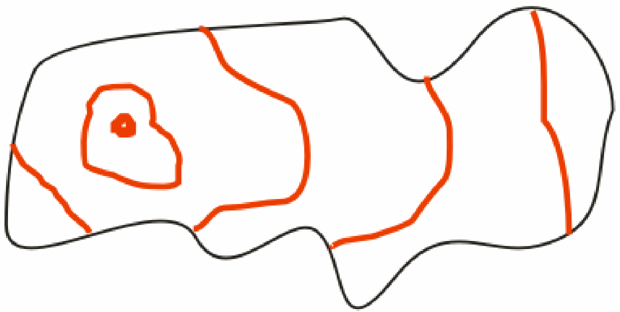
Potency or dislocation moment: $DS=DLW \propto L^3$

Seismic moment: $M_0 = \mu DS=\mu DLW \propto L^3$

Effective fault radius $R=(S/\pi)^{0.5}$

Key dimensional parameters
B. Kinematics

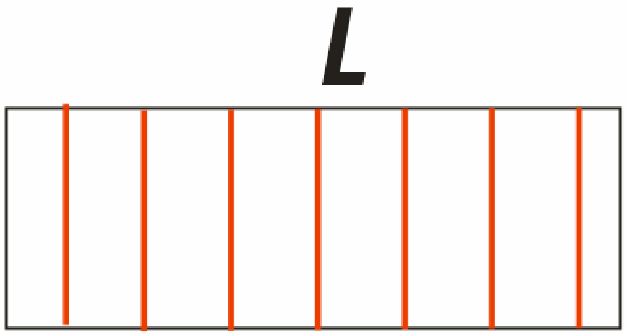
Rupture propagation history



Real case

$t_{front}=t_{front}(x,y)$

(isochrones)



Idealized case:

$t_{front}=v_{rup}\,x$

rupture duration T:

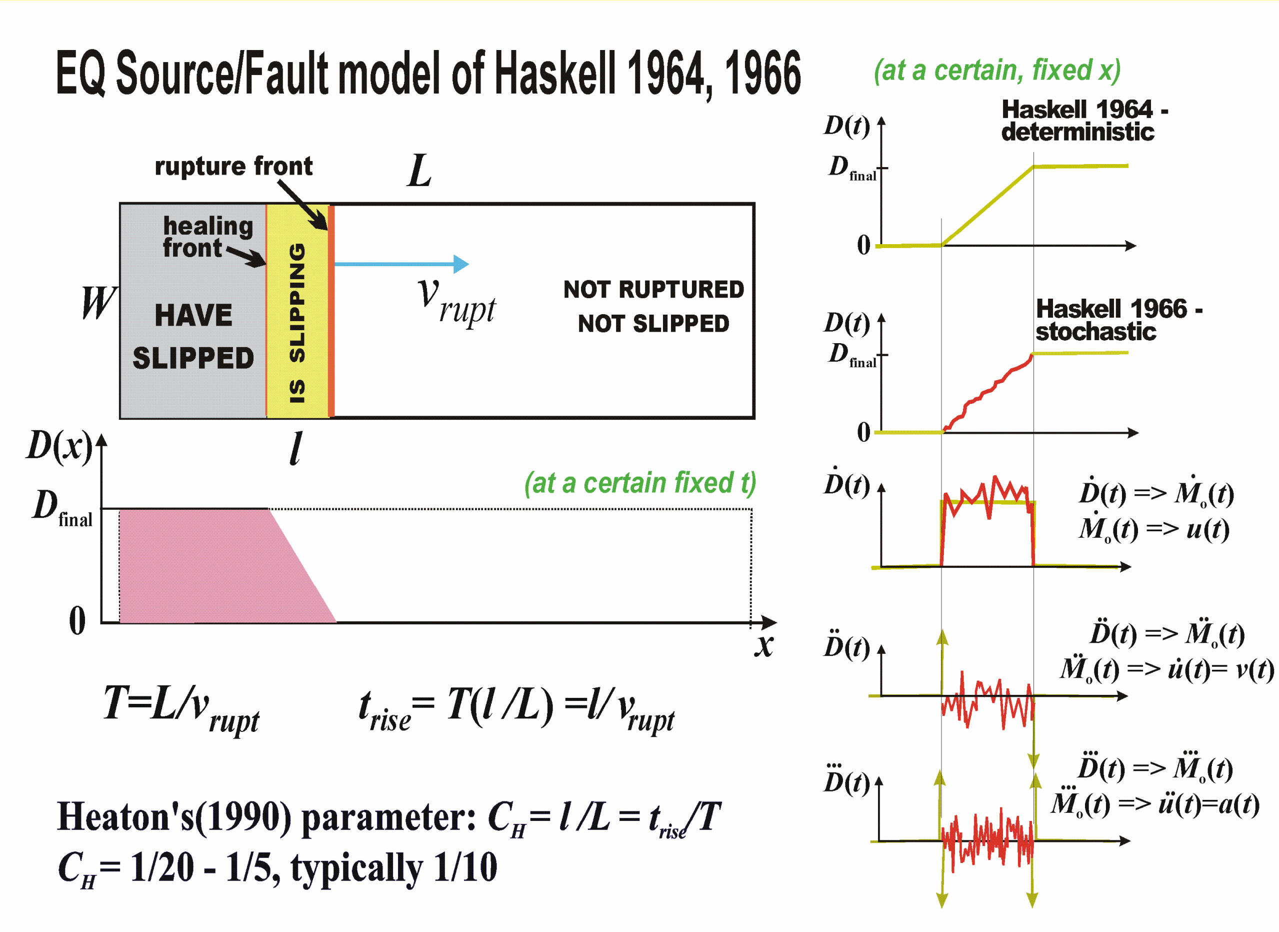
$T=v_{rup}\,L$

Magnutudes:

$10^{M}=A\propto$
$M_0/T\approx$
$\mu LWD/T=$
$\propto LD\propto L^2$
$\propto M_0^{2/3}$

M=

Key dimensional parameters
B. Local slip/dislocation formation



(Ideal case only)

Slip pulse width: l

Local slip formation time: $T_{rise} = T_r$

Local slip velocity

$v_{slip} = D/T_{rise}$

List of dimensionless or effectively dimensionless parameters and their typical values for natural tectonic earthquakes (average values over fault area)

[dropped coefficients on the order of 1]

Strain drop	$\Delta\varepsilon \approx \boldsymbol{D/W}$	10^{-4}-10^{-5}
Stress drop	$\Delta\sigma \approx \mu \boldsymbol{D/W}$	0.5-5 MPa [5-50 bar]
Stress drop	$\Delta\sigma \approx M_0/R^3$	0.5-5 MPa [5-50 bar]
Aspect ratio	$\boldsymbol{AR=L/W}$	1.5-3.5…..20
Mach number	Mach=v_{rup}/c_S;	0.5-0.9 global [up to 1.3 local]

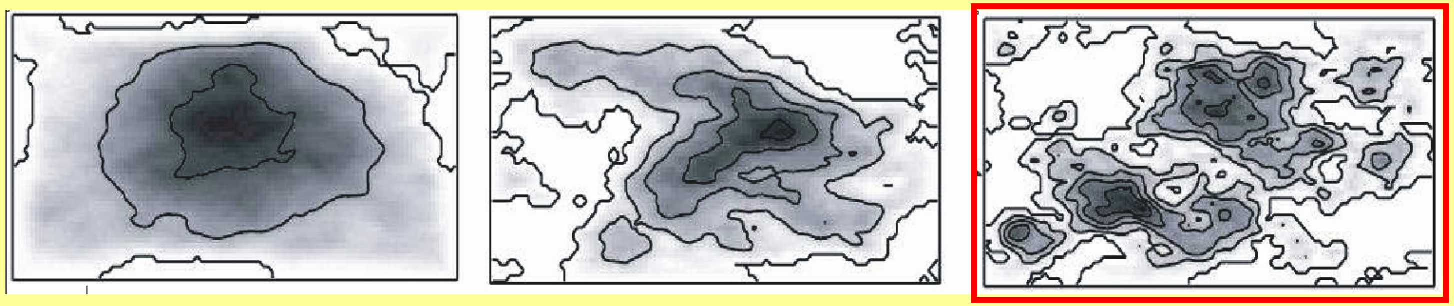
$c_S \equiv \beta$: *S* wave velocity

List of dimensionless or effectively dimensionless parameters and their typical values for natural tectonic earthquakes (2)

Relative width of slip pulse (=relative local rise time ***Trise/T***)	$C_H = l/\boldsymbol{L}$	0.1
Local/dynamic stress drop	$\mu \boldsymbol{D}/l$	30MPa [300 bar]
Local slip rate	$\boldsymbol{D}/\boldsymbol{T}_{rise}$	100 cm/s
Apparent stress drop	$\mu \boldsymbol{E}_{\text{seismic}}/\boldsymbol{M}_0$	0.5-5 MPa [5-50bar]

---------------------- less usual parameters ------------------------------

Coefficient of variation $(D^{0.5}(\cdot)/E(\cdot))$ of local stress drop field	$CV(D(x,y))$	1.0
Number of “asperities”	N_a	3



Slip maps D(x,y) obtained using various assumed wavenumber spectra, with various degrees of expression of “asperities”; the preferred case: $\boxed{}$

scaling / similarity: $\Delta\sigma$

Similarity with respect to strain of stress drop ($\Delta\sigma$) can be seen in data as empirical trends that follow predictions of dimensional analysis; or as scale-independence of empirical estimates of $\Delta\sigma$

In case of similarity $\Delta\sigma$ must be constant

[Conceptually, $\Delta\sigma$=const might follow from the assumption of scale independence of *ultimate strength* (or, merely, strength) of Earth material. However, the concept of *ultimate strength* is not quite clear in itself and to a large degree outdated.
Alternative concepts, like scale-dependent fracture toughness, have been tested, but no final consensus attained.]

Observed systematic stress drop/strain drop variations:

(1) Depth dependence *(the deeper, the stronger)*

(2) Distance from main plate boundary *(the farther, the stronger)*

(3) Return period of rupture on a particular fault segment *(the rarer, the stronger)*

Magnitude dependence of $\Delta\sigma$ is a matter of acute controversy:

1st team: $\Delta\sigma$=const at any M

2nd team: $\Delta\sigma$=grows with magnitude from M=1 to M=5-6;

and stable at M=6+

Consensus not seen

scaling / similarity: Mach

Similarity with respect to Mach$=v_{rup}/c_S = L/T$:
In case of similarity Mach ***must be constant***

no significant deviation of *fault-average* (“global”) Mach from typical values **Mach**=**0.7±0.2** was noticed.

However, very significant *local* variations of Mach, with many examples of **“supershear”** rupture with Mach>1 over large sections of entire fault, has been discovered, mostly in the last 15 years.

Some scaling or, really, similarity: AR

Similarity/scaling with respect to AR=L/W :

(1) Two classes of sources with different trends:

- "short" faults with AR=1-4, mostly dip-slip ones ("s")

Examples: Tohoku 2011(subduction), Northrige 1994 (crustal),

- "long" faults with AR>4, up to 20; mostly crustal strike-slip ones.("*l*")

Examples: San-Francisco 1906 (crustal), Sumatra 2004 (subduction)

(2) Within each class, clear magnitude dependence:

Low AR≈1-1.5 for M=4-5; increase to big values like 3-4 for "s" class and like 20 for "*l*" class

scaling / similarity: AR (2)

============= *Probable explanation for magnitude dependence:* ============

The larger is magnitude, the larger is fault area and the more are chances that it will be elongated to fit in the brittle zone of limited width.

=============*Probable explanation of 2 classes .* ========================

External non-uniformity spoils scaling. Division into classes is created by various brittle zone size along rupture propagation direction (vrup vector).

Crustal event case, brittle zone width 10-20 km, defines max W;

dip-slip: L is limited /segmentation; AR around 1-3
strike-slip: L is not limited, AR up to 20

Subduction event case mostly dip-slip , brittle zone width 50-200 km, defines max W

L is not limited, W is limited by 50-200; AR rarely above 4, sometimes up to 10-15

Probable case of lack of similarity in scaling : $C_H = l/L$

Similarity/scaling with respect to relative width of slip pulse (=relative local rise time).

$C_H = l/L = T_{rise}/T$

In case of similarity C_H must be constant

Weakly studied field. For l, very limited amount of direct measurements.

To a large degree, C_H reproduces the ε parameter – the degree of "partial stress drop" - proposed by Brune (1970)

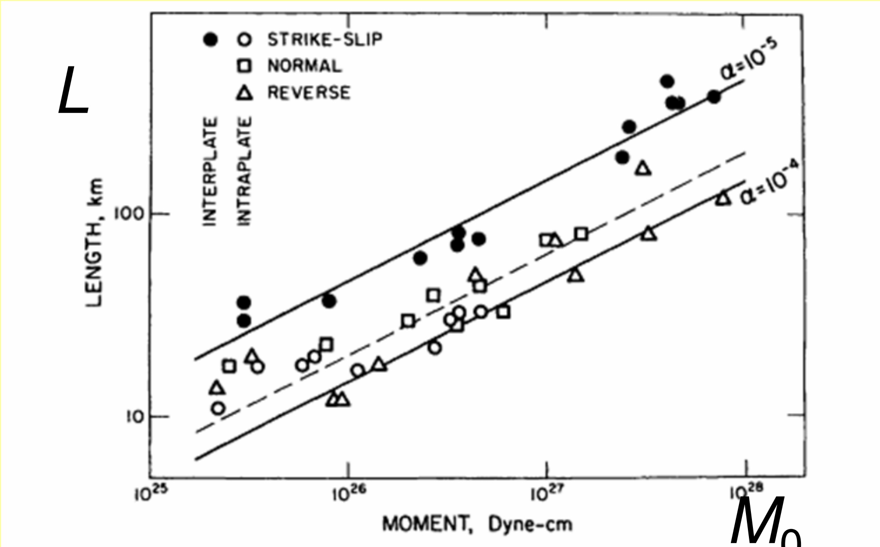
Gusev 2013 proposed that one can estimate T_{rise} from the second corner frequency as seen on the source spectrum; this point to be described in detail further.

If this works, empirical data suggest that C_H is clearly non-constant; similarity breaks:

T_{rise} seems to scale approximately as $T^{0.5}$; and therefore C_H as $T^{-0.5}$ or $M_0^{-1/6}$

Supporting evidence

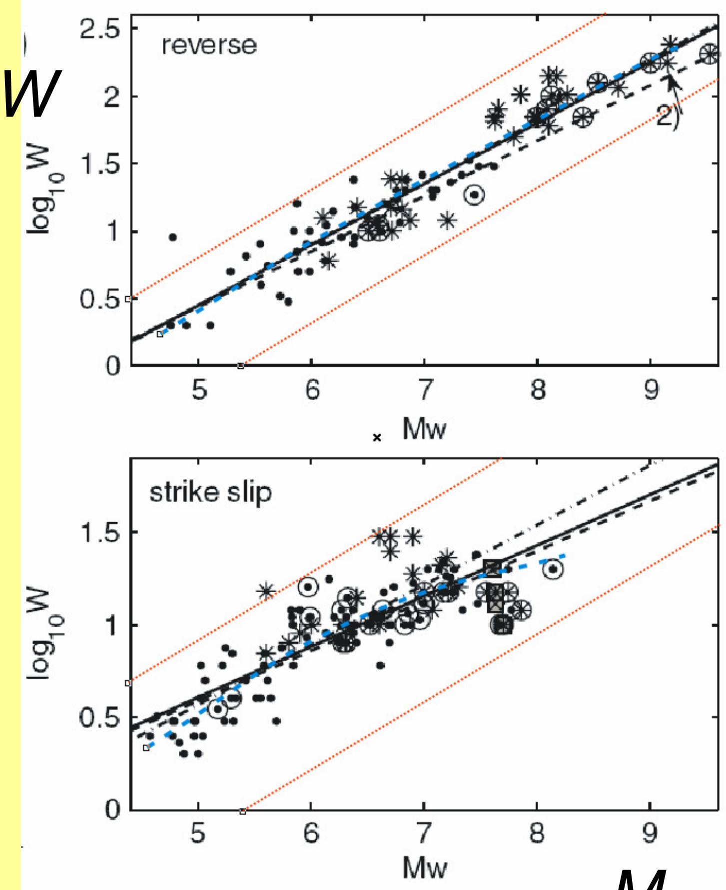
L



M_0

FIGURE 9.26 Fault length versus moment, showing a clear separation of intra- and inter-plate earthquakes. (From Scholz *et al.*, 1986.)

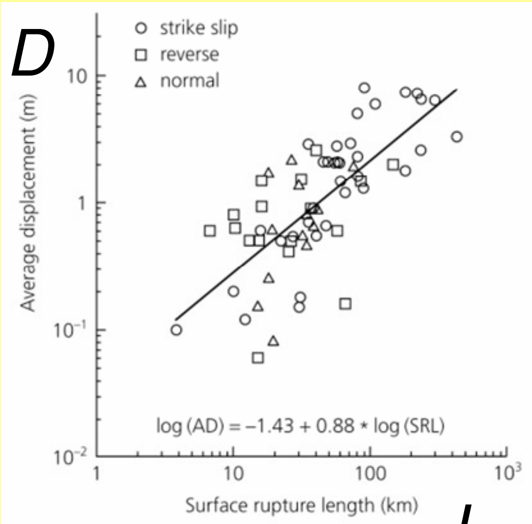
W



Blazer et al.2010

M_w

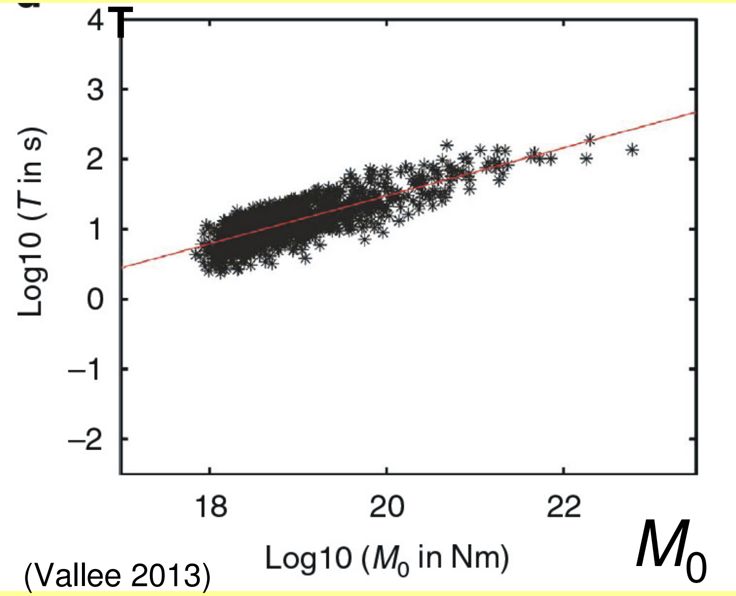
D



L

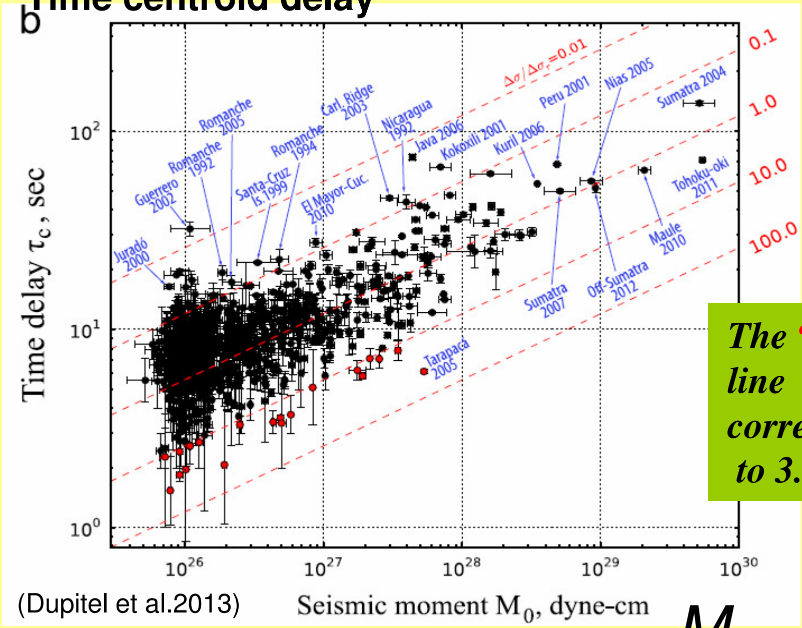
Supporting evidence

Duration, T



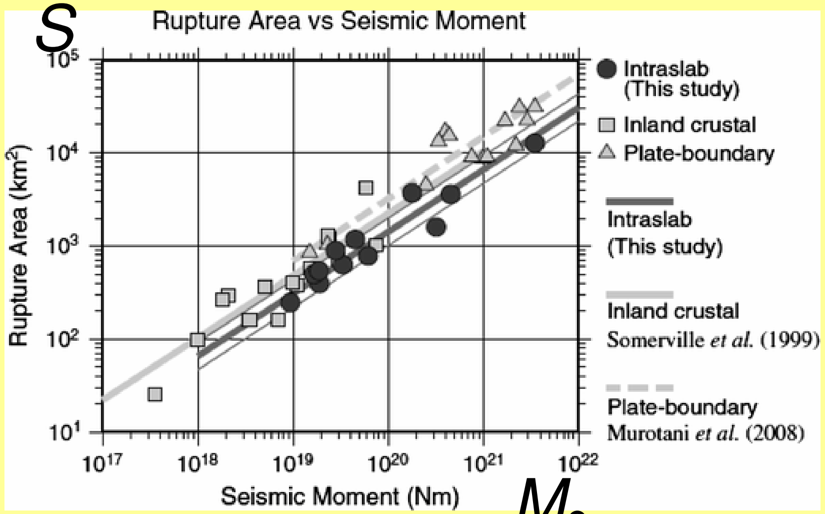
(Vallee 2013)

Time centroid delay



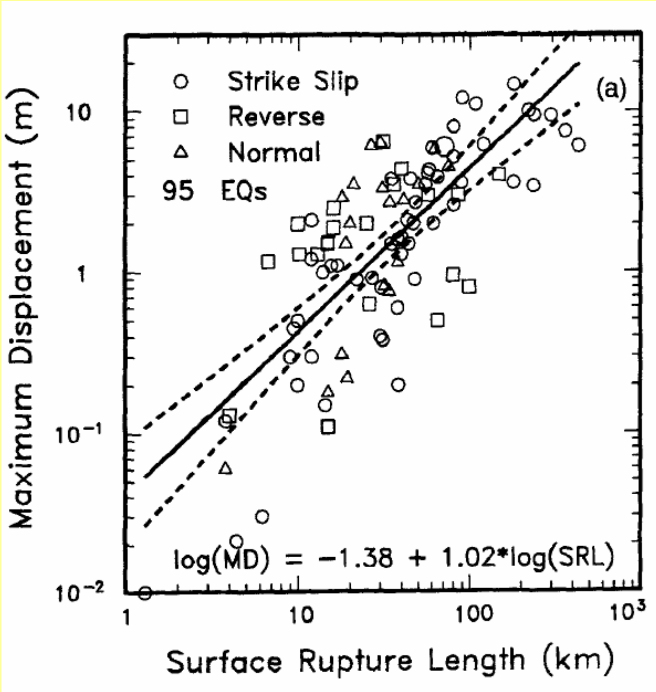
(Dupitel et al.2013)

The "1.0" line corresponds to 3.7 bar



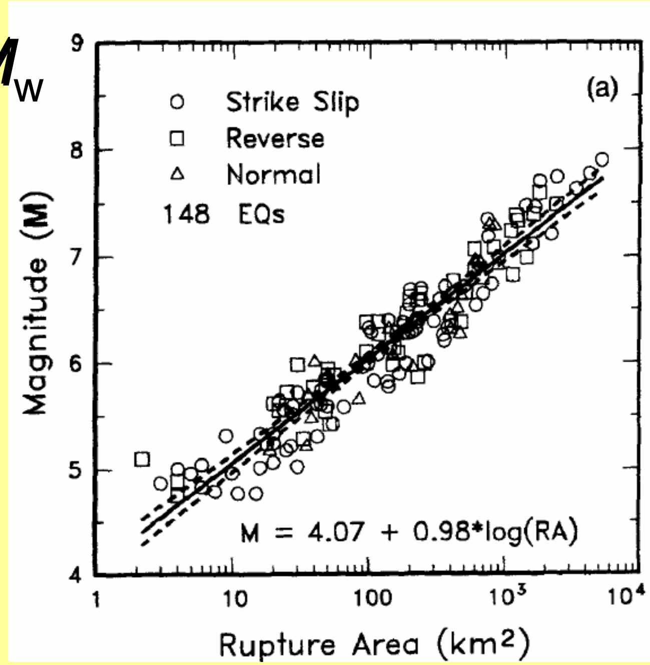
(Iwate&Asano 2011)

Supporting evidence



L

M_w



94)

S

(Wells&Coppersmith1994)

ЧАСТЬ 2: СПЕКТРЫ

Подобие и скейлинг для характерной частоты
Scaling and similarity for characteristic frequency

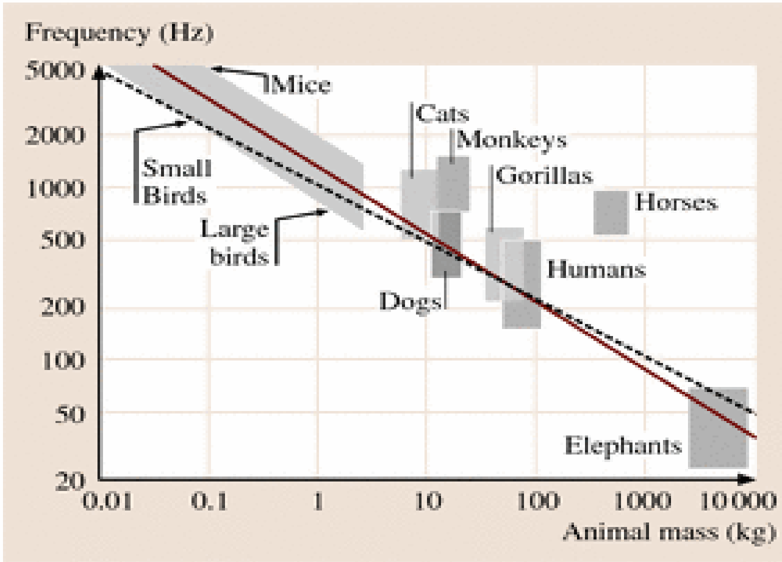


Fig. 19.1 The frequency ranges of the emphasized frequencies of vocalization in a large range of land-dwelling animals, plotted as a function of the mass of the animal. …egression $f \propto M^{-0.33}$ while …n $f \propto M^{-0.4}$, as discussed in

$f_{fund} \sim L^{-1}$
$\sim MASS^{-1/3}$

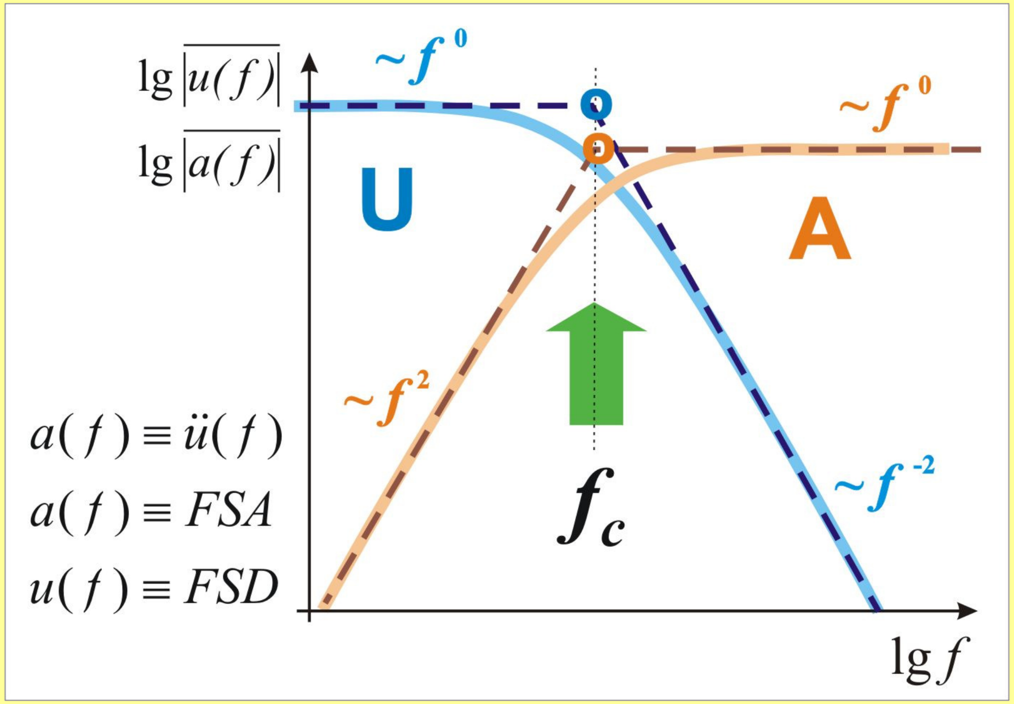
Подобие
Similarity

$f_{fund} \sim$
$MASS^{-0.4}$

Скейлинг есть;
но подобие нарушено
Scaling is present; no similarity

"ϖ^2" or "omega-square" spectral model of far-field earthquake source radiation, after Aki 1967 and Brune 1970: scaling with similarity

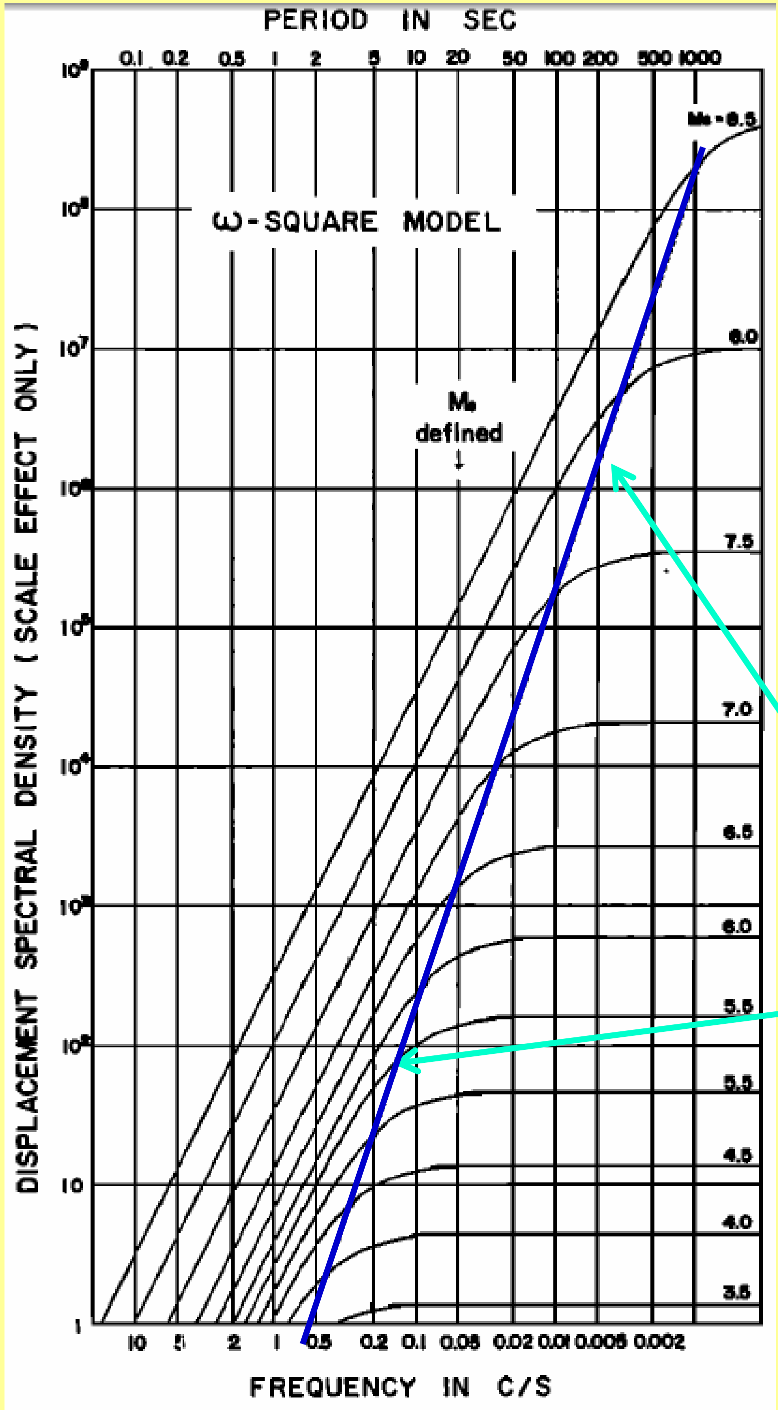
([Brune 1970] in the standard version of ε=1)



Single far-field displacement u(f) spectrum

Features:

1. $u(f=0) \propto M_0$
2. Single corner at $f_c \approx 1/T$
3. ϖ^2 or f^{-2} HF branch of $u(f)$; thus, flat $a(f)$ spectrum



Scaling Law of Seismic Spectrum (Aki 1967)

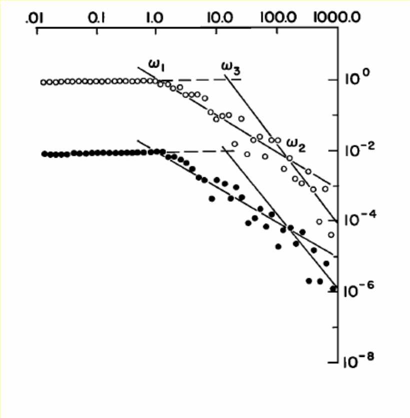
The family of far-field displacement spectra $u(f|M_0)$

Assuming similarity

$f_c \propto M_0^{1/3}$

Brune 1970 version of “omega-square” spectral model and its later practical implementation

- fault description deterministic, not stochastic after Aki 1967
- explicit formulas relate M_0, f_c, $\Delta\sigma$ and R
- in case of more complicated spectral shapes:
  (1) “empirical-asymptotic” HF branch is permitted to have slope in the 1.0-2.5 range; and / or
  (2) complications around the corner are ignored, and “intersection of asymptotes” is taken as ***the*** corner frequency



This picture from Savage 1972 was not a recommendation, only statement of a problem. However, many spectral studies actually used it in this way, taking “ω_3” as the empirical corner position

This is the probable reason why spectral and inversion estimates of Ds often do not match (typically 30 bar against 5-15 bar)

On the nature of spectral corners in deterministic source models.
Why stochastic fault model is a must?

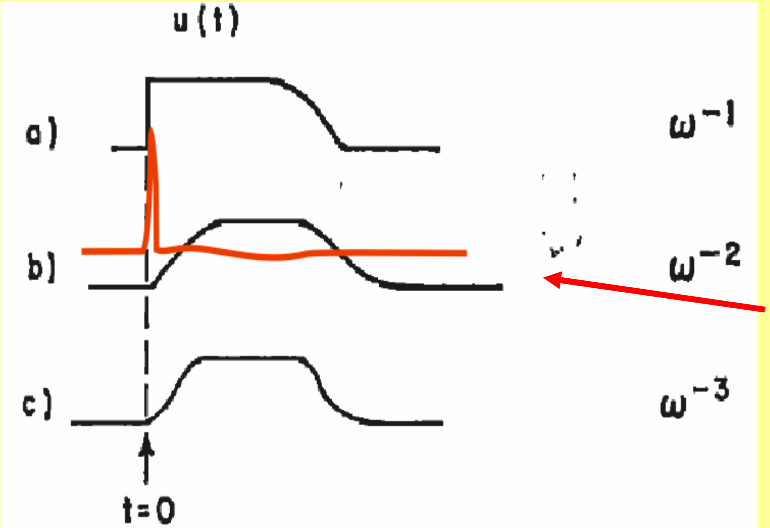


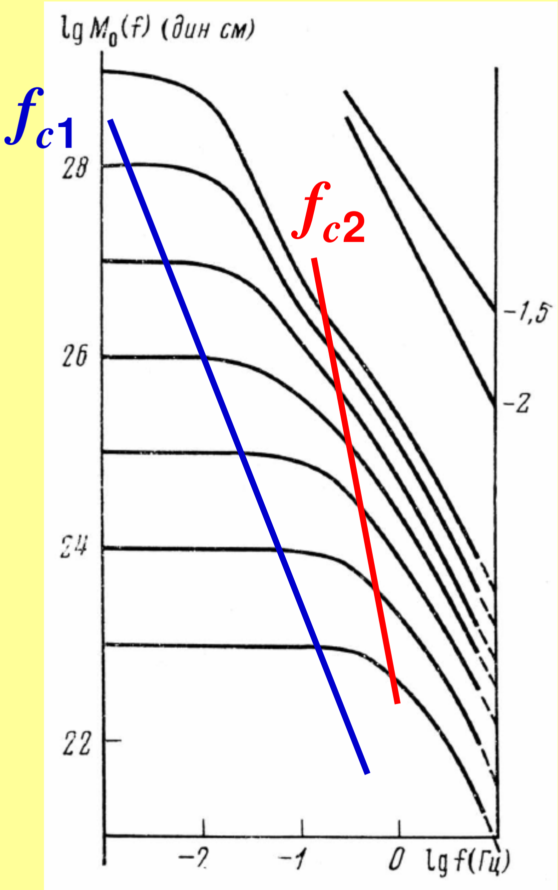
Fig. 4. The relation between the pulse form and the asymptotic behavior of its spectrum [after *Bracewell*, 1965]. Curves *a*, *b*, and *c* exhibit discontinuities at $t = 0$ in the displacement u, velocity du/dt, and acceleration d^2u/dt^2, respectively.

This picture from Savage 1972 illustrates how discontinuities in the displacement waveform, (or source time function, STF) are related to emergence of spectral corners.

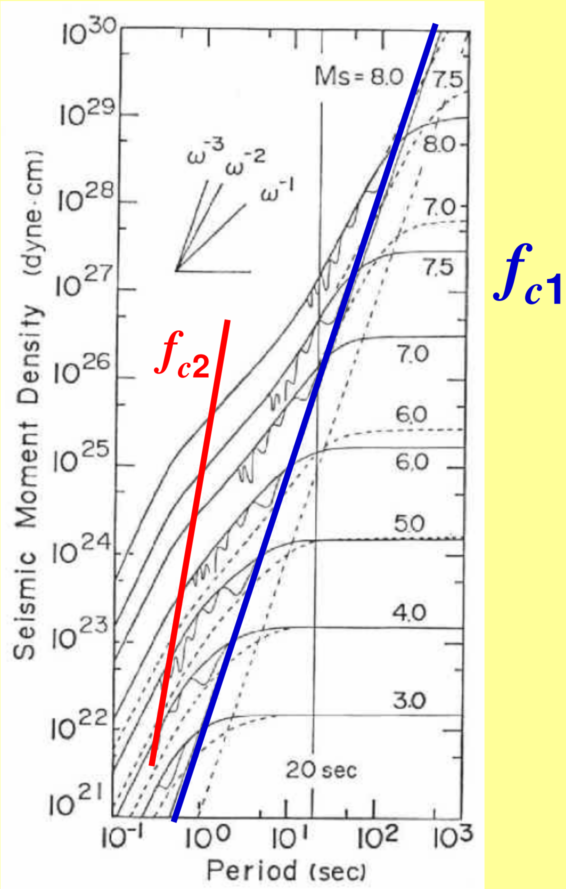
The preferred waveform **b** with angular point and thus with "ω^{-2}" spectrum, after passing to acceleration, produces delta-like acceleration spike (red trace, my), nothing common with realistic noise-like accelerogram.

The standard textbook STF of trapezoidal shape produces precisely four spikes. Therefore, deterministic models give no hope in creating realistic broadband source model. Stochastic models can help.

First attempts for more realistic spectral families: strict similarity is rejected



(Гусев 1979)
(Gusev 1979)



(Takemura&Koyama1982)

Key feature of the new generation of spectral models is the lack of similarity that was needed to describe real earthquke data

Revised (1978-1981) spectra remind Aki1967 spectra at M=5,

And show two-corner or even two-hump shape at M=8

More advanced spectral families with no similarity

(Gusev 1983)

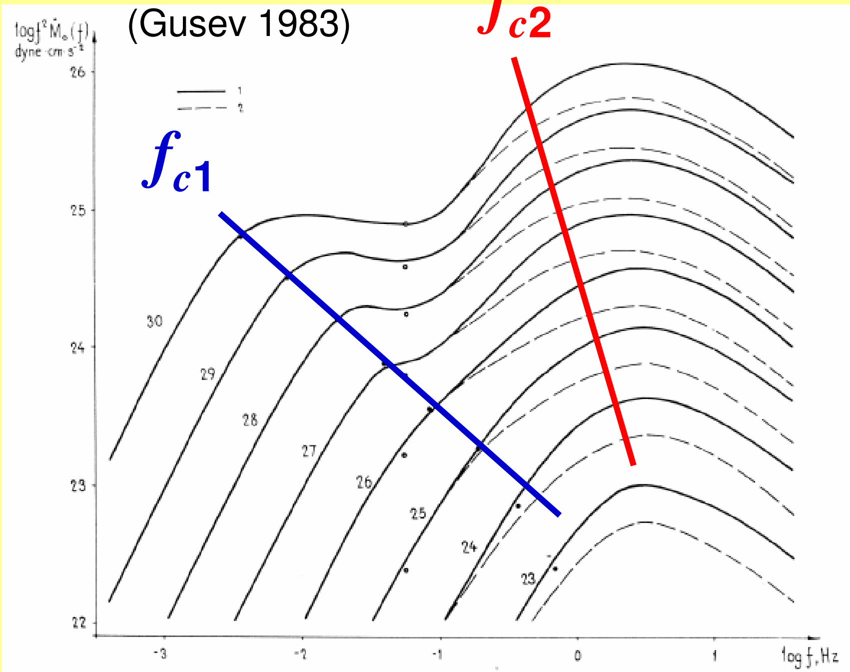


Figure 3. A set of average spectra of value $f^2\dot{M}_0(f) = (2\pi)^{-2}\ddot{M}_0(f)$ for log M_0 = 23–30. Continuous lines: spectra as observed at the Earth's surface, broken line: reduced to the source. Points: spectral level at corner frequency according to the model $\dot{M}_0(f) = M_0[1+(f/f_0)^{\gamma}]^{-1}$. Circles: spectral level derived from $M_{LH}(M_0)$ curve.

(Papageorgiou&Aki 1983,1985) PAGEOPH,

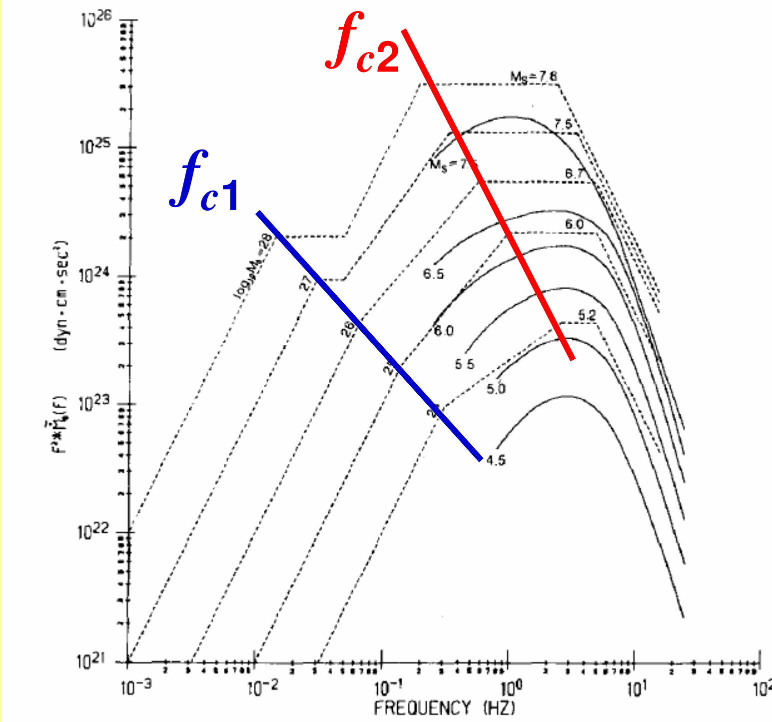


Figure 4

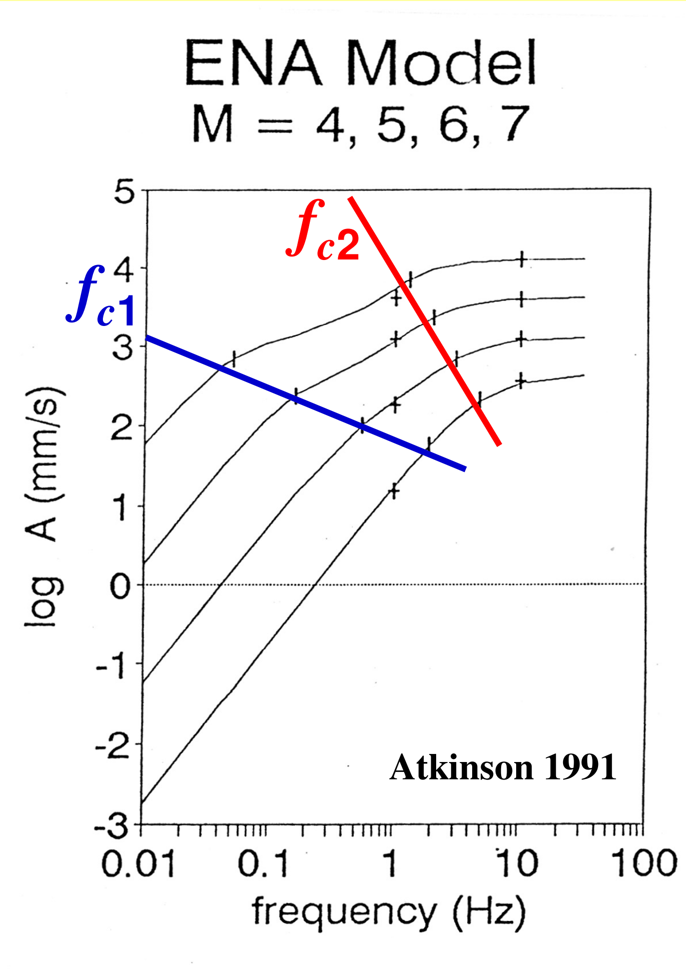
acceleration source spectra we constructed using the observed parameters of the specific barrier model (intermittent line), is compared with Trifunac's (1976) empirical spectra.

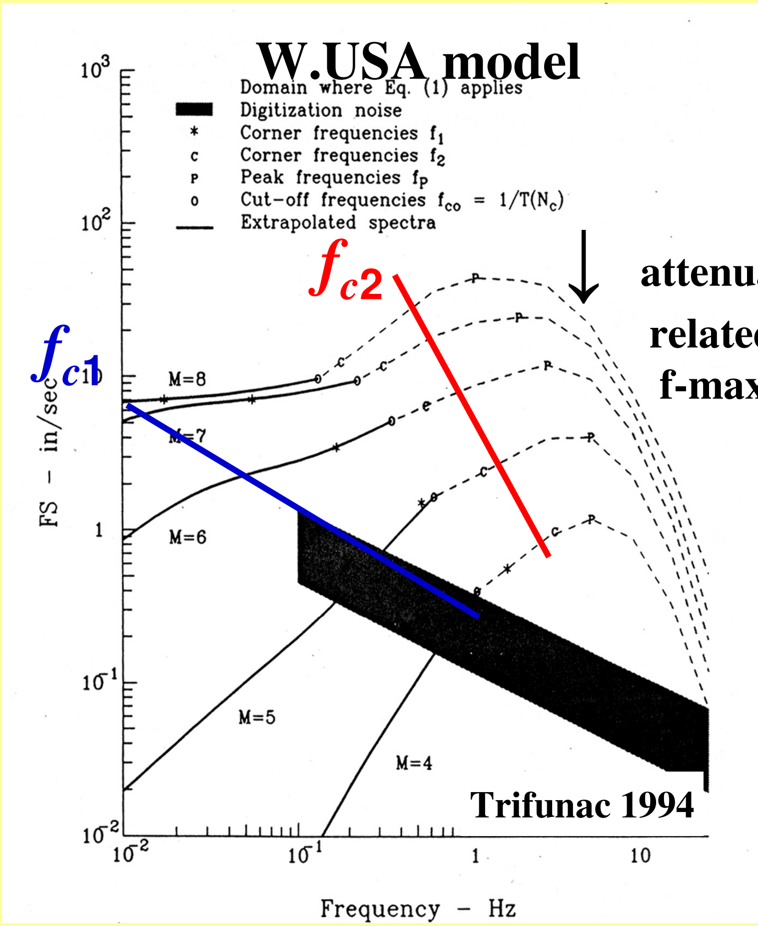
Shown are "acceleration source" spectra, those reflecting acceleration source time function.

Decay of these spectra at HF part are related to the assumption of "source-controlled f-max" .

Its role was greatly overestimated at that moment; still it can often be recovered from data, and this assumption reflects reality.

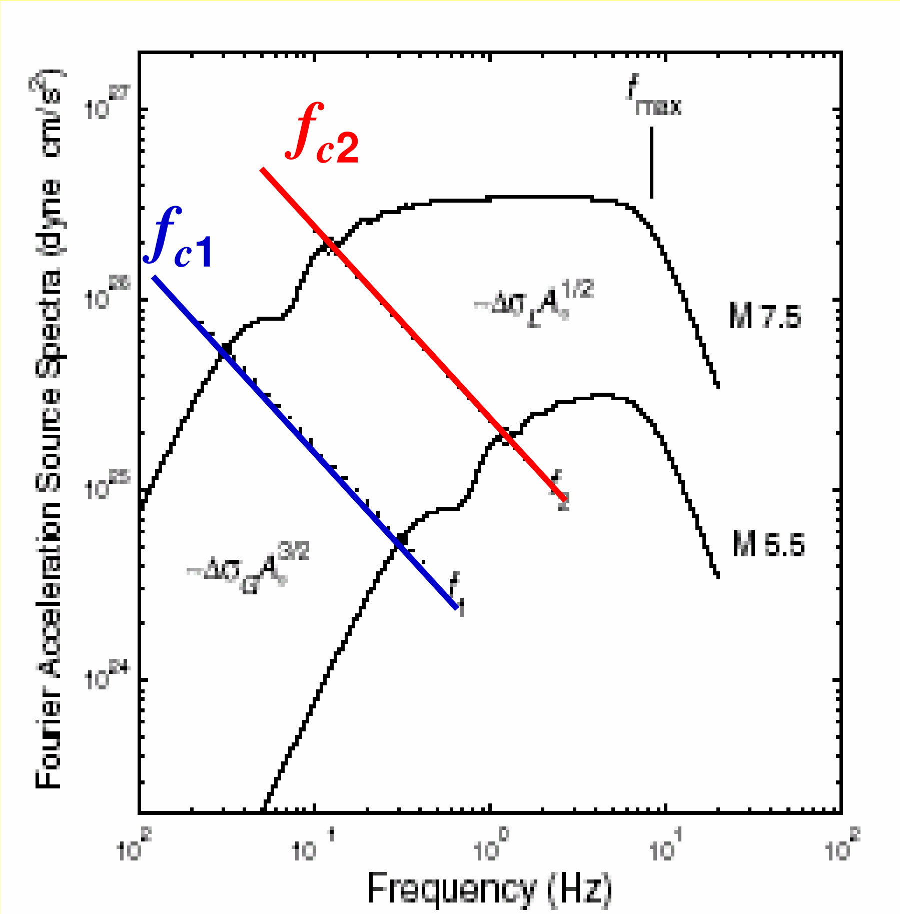
Empirical spectral scaling laws with flat *accelerogram* spectra approximating *source* acceleration shapes





attenuation-related f-max

Recommended spectral scaling laws (Halldorsson&Papageorgiou 2005)

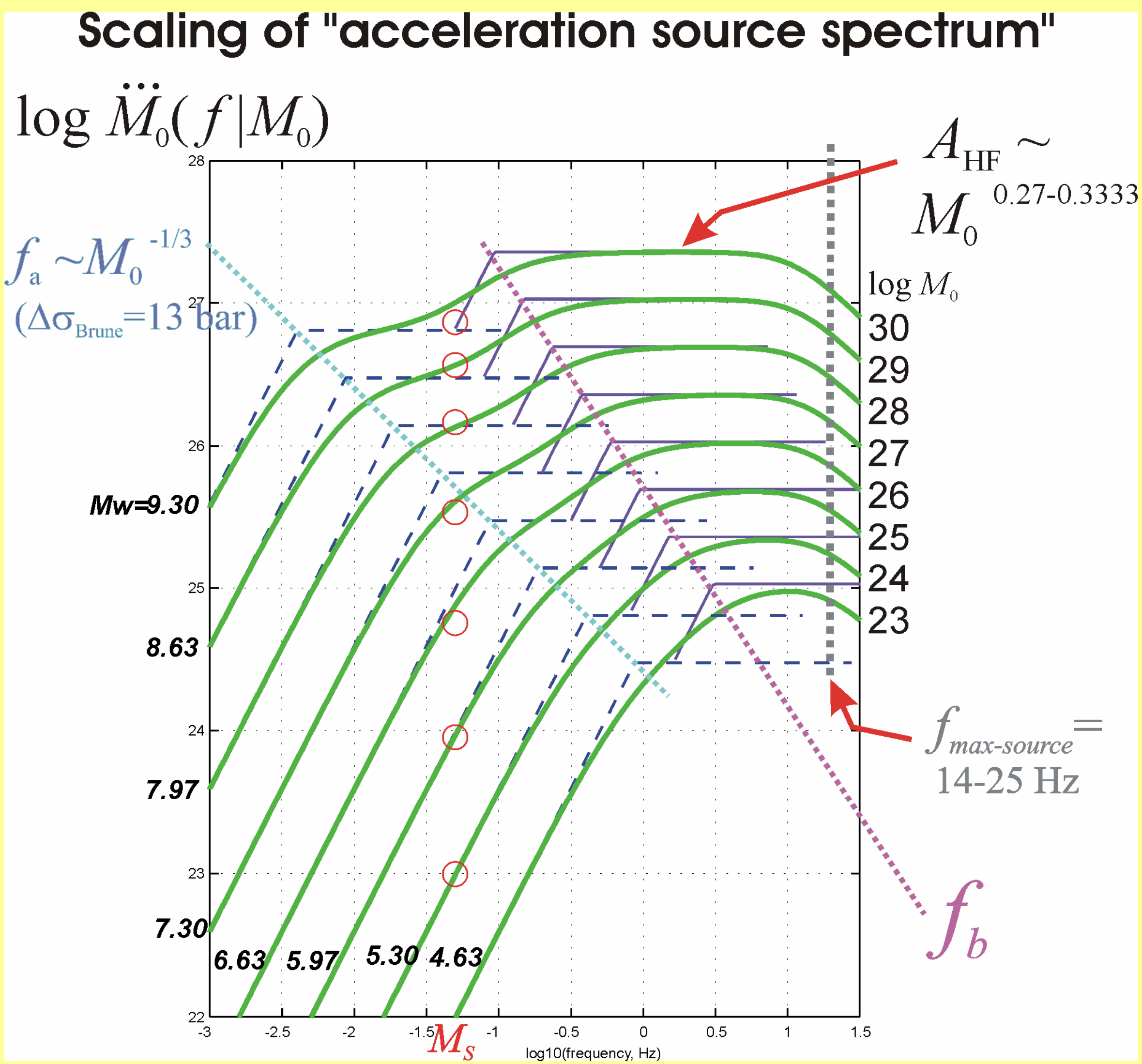


(Halldorsson&Papageorgiou 2005)

f_2 is definitively present

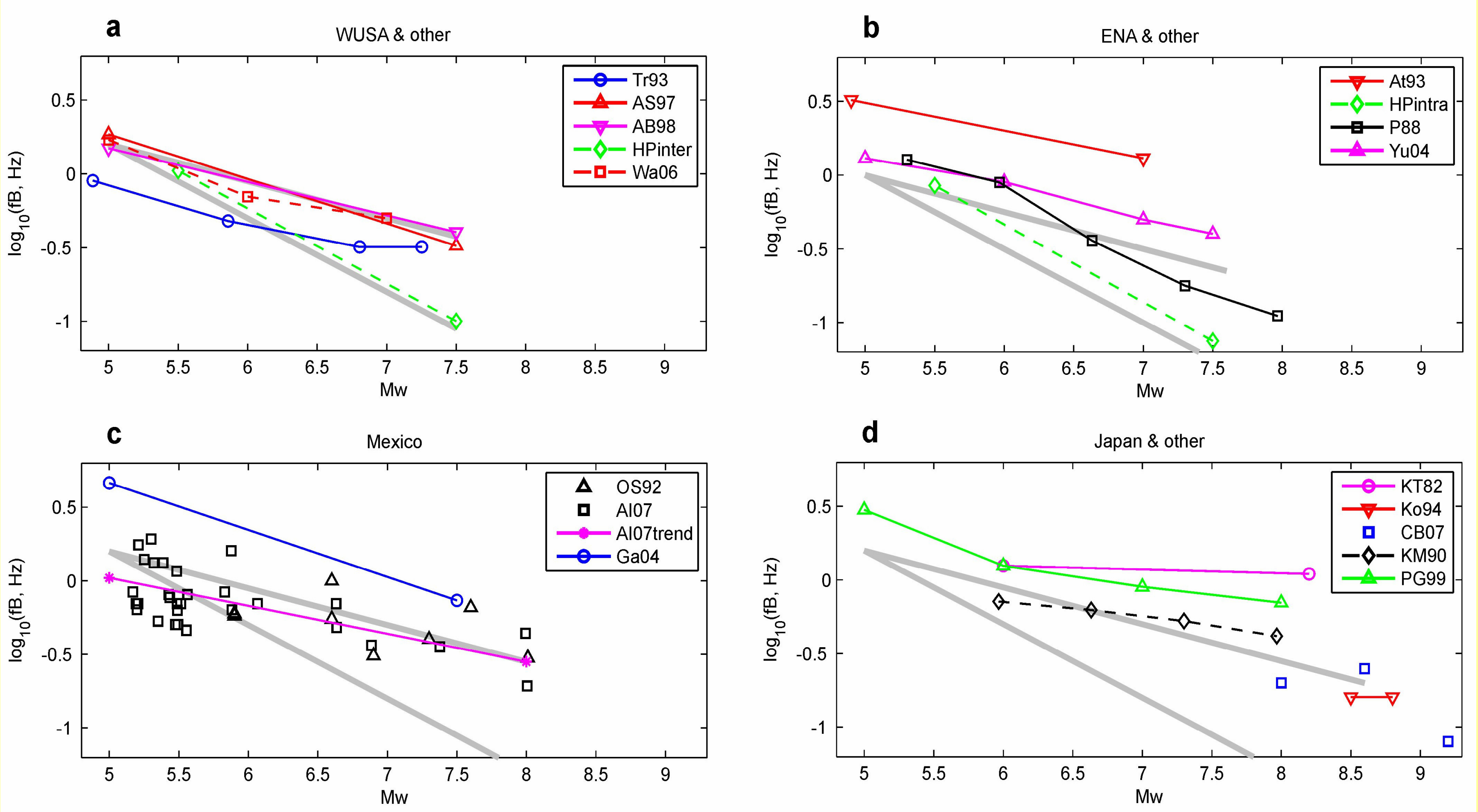
but it is correct to assume it to scale as f_c ???

Work version of Gusev 2007 (unpublished)



Compilation of $f_{c2}(Mw)$ scaling:

$$f_{c2} \propto 10^{0.25\text{-}0.30Mw} \propto M_0^{0.17\text{-}0.2} \propto f_{c1}^{0.5\text{-}0.6}$$

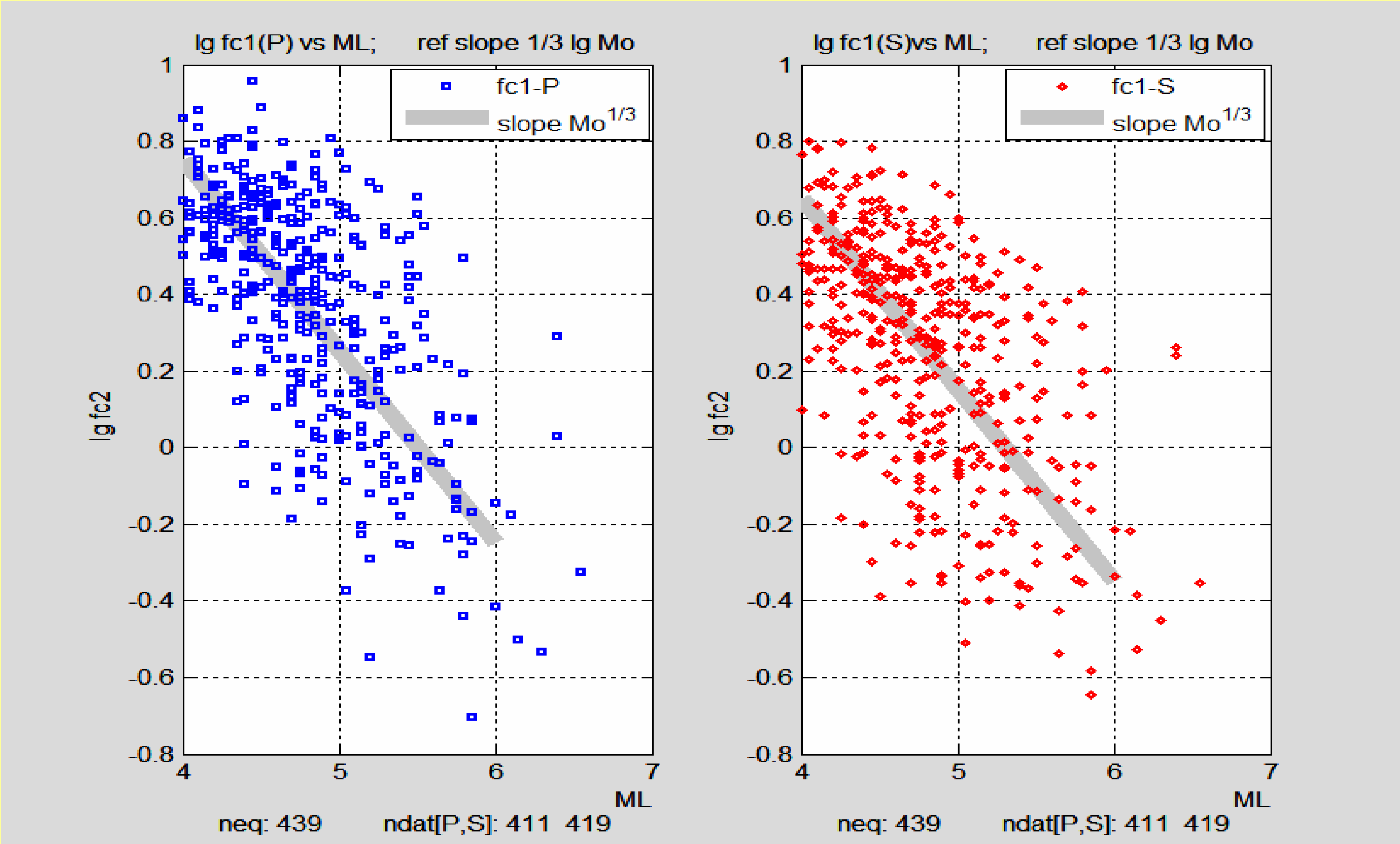


Kamchatka data (Gusev,Guseva 2014).

f_{c1}(M): $\mathrm{dlgf_{c1}/dlgM_0} \approx -1/3$

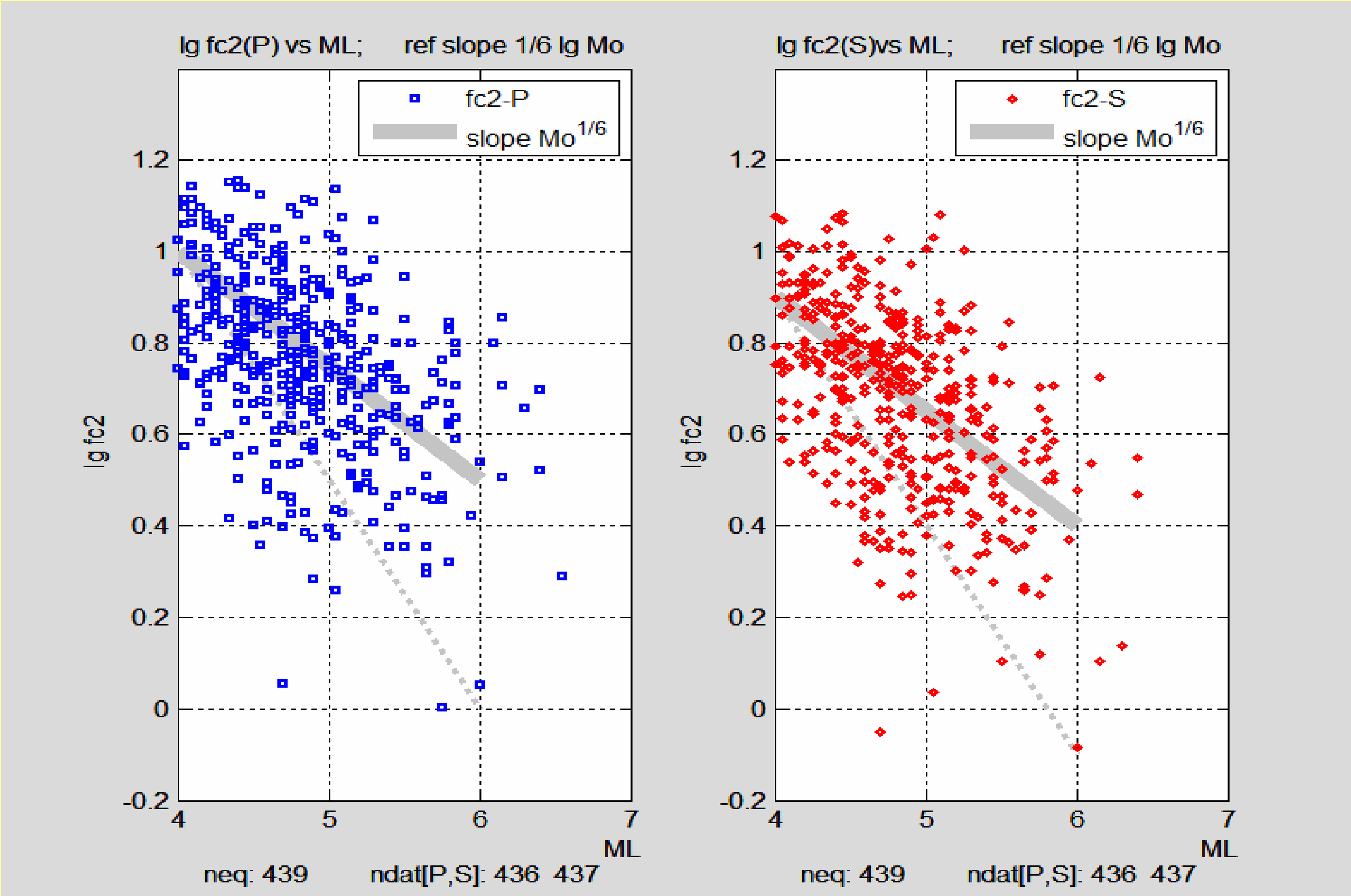
common, regular trend;in agreement with the similarity concept

скейлинг в согласии с идеей подобия



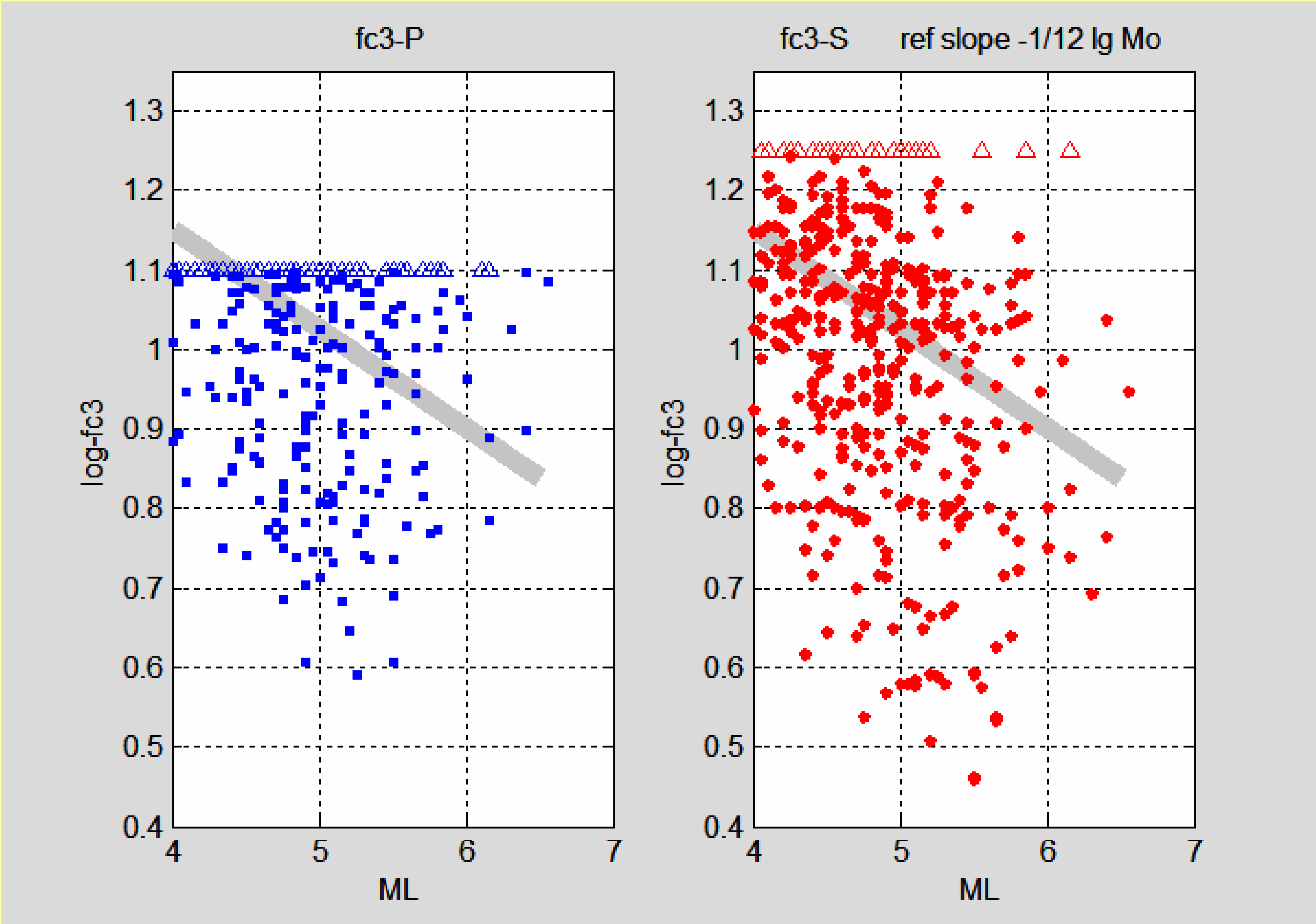
f_{c2}(M): $d\lg f_{c2}/d\lg M_0 \approx 0.15\text{-}0.18\ [\pm 0.011] \ll 1/3$

подобие явно нарушено   similarity is broken

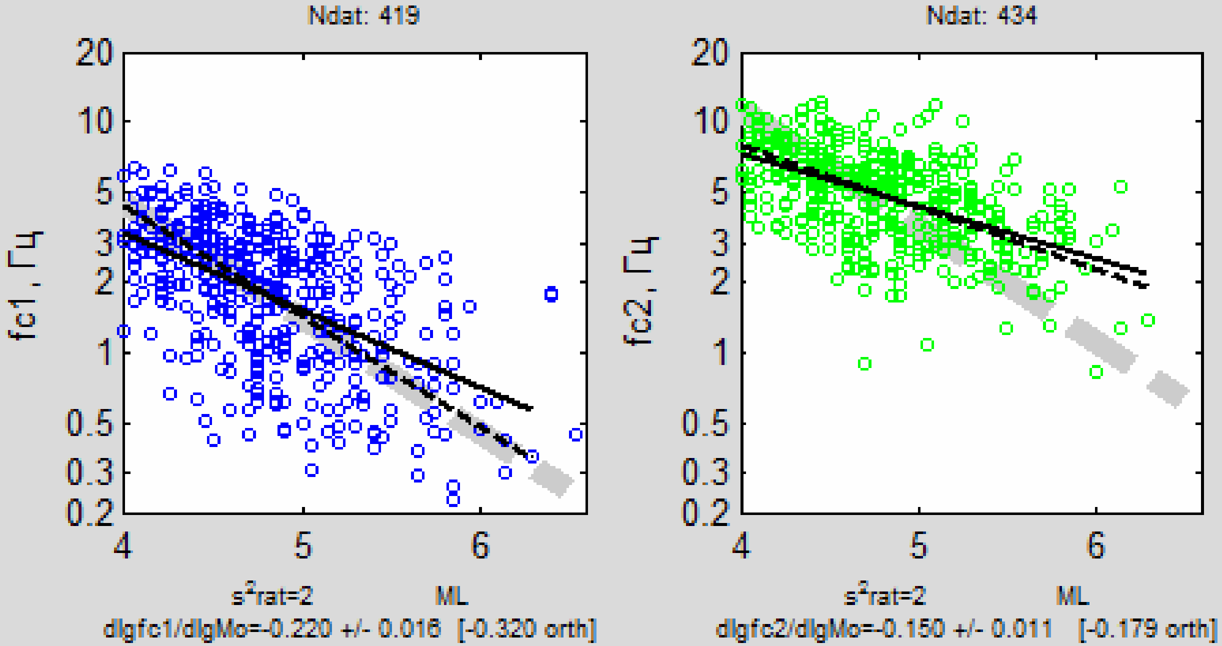


f_{c3}(M): $dlgf_{c1}/dlgM_0 \approx -0.08 \pm 0.013$

подобие грубо нарушено similarity is broken blatantly



All three trends side by side (S-wave)



FIN

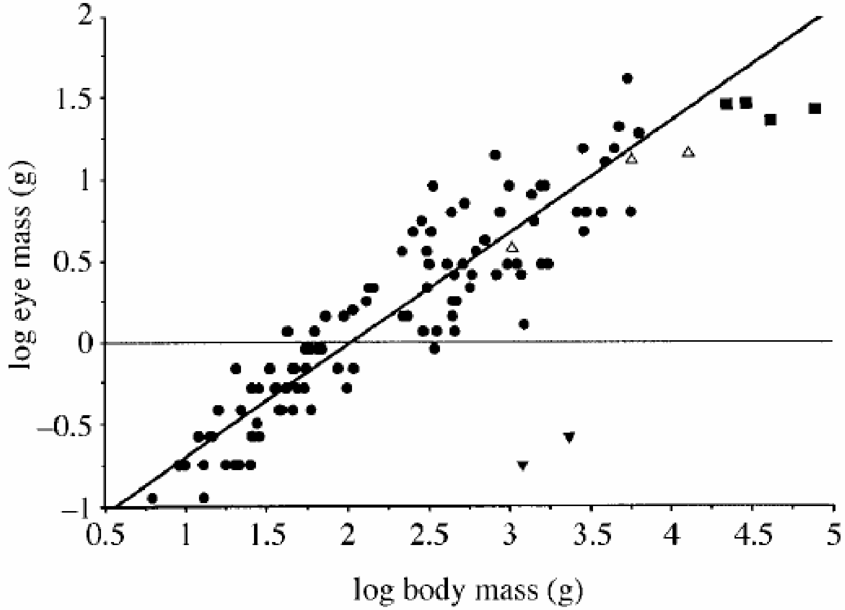
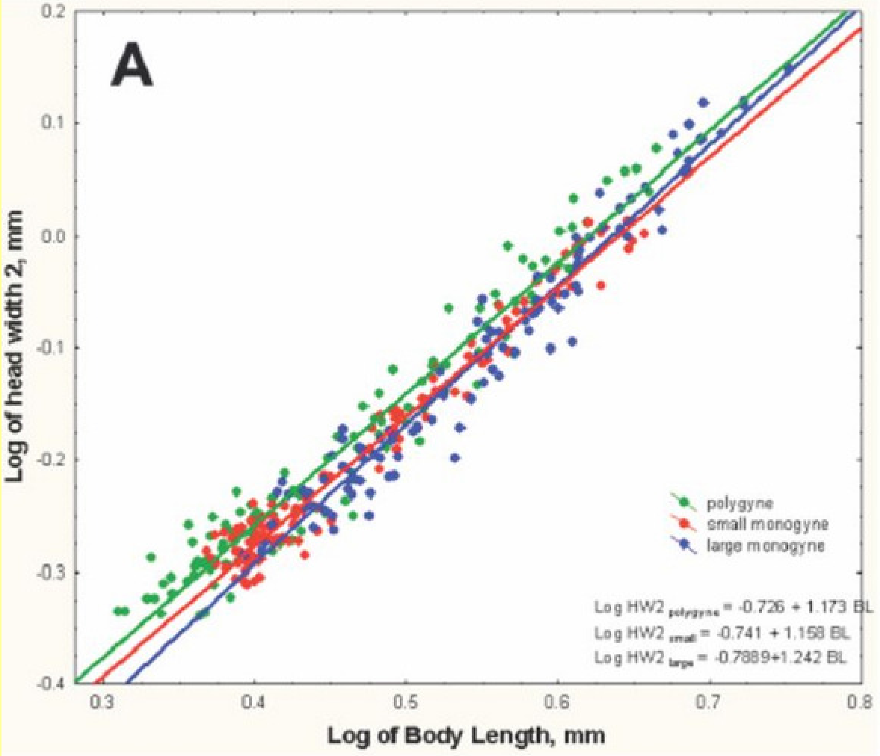


Figure 1. The regression of eye mass on body mass for 104 species of flying birds, each from a different family. $y = 0.682x - 1.379$, $r^2 = 0.846$ and $p < 0.0001$. Slope of reduced major axis regression = 0.741. Although not included in the



the fire ant *Solenopsis invicta* workers (A)

Although both polygyne and monogyne colonies displayed positive allometry in head width above the eyes, workers from large monogyne ants had a higher growth rate (1.23, as opposed to 1.16 for small monogyne and 1.17 for polygyne) in relation to the total body size.

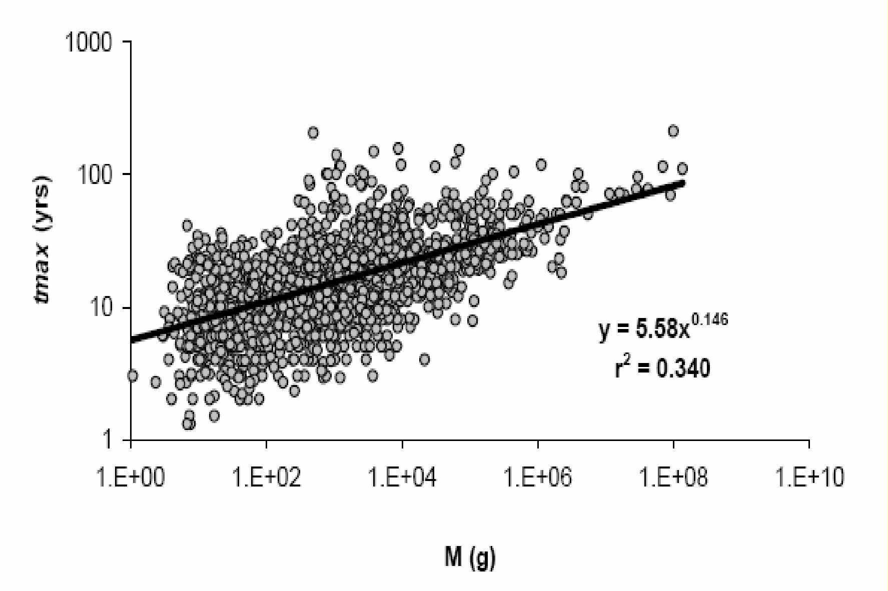


Figure 1: Correlation between maximum lifespan (*tmax*) and typical adult body mass (*M*) using all species (n = 1,701) present in AnAge build 8. Plotted on a logarithmic scale.

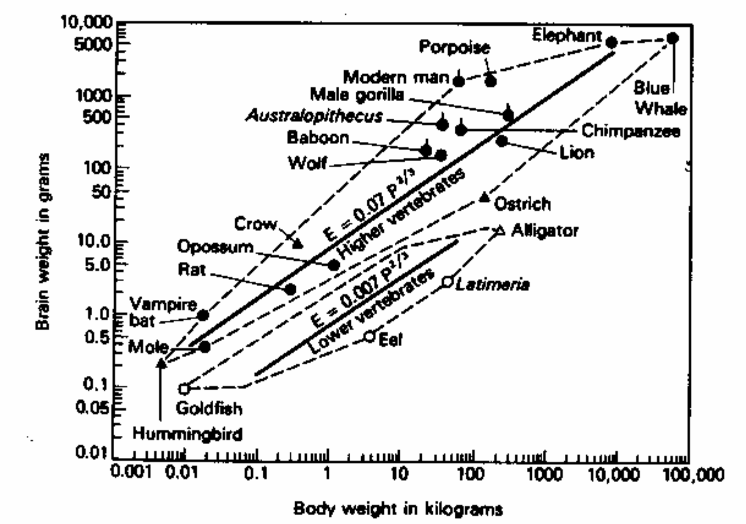


Figure 2. Brain weight as a function of body weight (from Jerison, 1973; see

Principles for analysis of scaling (2)

Generally, scaling analysis assumes that no intrinsic dimensional (spatial, temporal, etc) scales exist within the problem under study.

When a relevant dimensional parameter appears, scaling can often identify it, then power law is violated and critical size shows itself in a scaling diagram as an anomaly.

Spectral peak in a spectrum on the background of white noise or power-law behavior is a standard example.