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GEOPHYSICS

Characteristic Scale of Heterogeneity of Seismically Active Fault and Its Manifestation in Scaling of Earthquake Source Spectra[1]

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Abstract—Previously, similarity of source spectra of Kamchatka earthquakes with respect to the common corner frequency f_{c1} and the expressed deviations from similarity for the second f_{c2} and the third f_{c3} corner frequencies were revealed. The value of f_{c3} reflects the characteristic size L_{is} of fault surface; correspondingly, $L_{is} \approx v_r T_{is}$, where v_r is the rupture speed and $T_{is} \approx 1/f_{c3}$ is characteristic time. The estimates of f_{c3} are used for normalizing f_{c1} and f_{c2}. In this way one obtains dimensionless rupture temporal parametres τ_1 and τ_2 and can further study the dependence τ_2 (τ_1). The growth of a rupture is considered as a process of aggregation of elementary fault spots of the size L_{is}. The dimensionless width of the random front of aggregation is on the order of τ_2. The relationship τ_2 (τ_1) approximately follows power law with exponent β. The estimates of β derived from earthquake populations of Kamchatka, USA and Central Asia (β = 0.35–0.6) agree with values expected from the known Eden's theory of random aggregation growth and from its generalizations.

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Recently for hundreds of moderate Kamchatka earthquakes the scaling of characteristic (corner) frequencies of source spectra has been studied [1]. Approximate similarity of the sources with respect to the usual corner frequency f_{c1}, and, along with it, the expressed deviations from similarity for the second f_{c2} and the third f_{c3} corner frequencies has been revealed. The probable meaning of differences between scaling of f_{c1}, f_{c2}, and f_{c3} will be discussed. We believe, as usually, that the earthquake source process is a sliding of walls of a seismically active geological fault that occurs within its certain area. It is supposed [2–4], that there exists the characteristic size of heterogeneity, denoted L_{is}, which is inherent for the fault surface. It manifests itself through the presence of high-frequency (HF) cutoff of source acceleration spectrum, i.e., "source-controlled f_{max}," further called f_{c3}. The relationship between f_{c3} and L_{is} is formed in the following way. Let us introduce "characteristic time" parameter T_{is}: during T_{is} the front of the running rupture passes just the distance $L_{is} \approx v_r T_{is}$, where v_r is rupture velocity, on the order of 3 km/s; we believe that $T_{is} = T_{c3} = 1/f_{c3}$. As regards times $T_{c1} = 1/f_{c1}$ and $T_{c2} = 1/f_{c2}$ we will believe that T_{c1} is close to the duration of rupture, T; and that T_{c2} is close to rise time T_r (local time of sliding at a certain fault point) [5].

In [3, 4, 6, 7], a few mutually supplementing hypotheses are put forward about the nature of f_{c3} and L_{is}. The value of L_{is} can be related to: (1) the thickness of the zone of the damaged rock that surrounds a fault; (2) the lower limit of the range of the sizes (wavelengths) of roughness over a fault surface; (3) the width of cohesion zone at the tip of a crack-like source; (4) the size of the source of a smallest earthquake; (5) the size of nucleation zone of a rupture. Anyhow, L_{is} is the size of a unitary seismically active spot on a fault. For the smallest earthquakes f_{c1} and f_{c3} approach and at last coincide with each other, and also with f_{c2} [2].

The observed slow decrease of f_{c3} with increasing earthquake seismic moment M_0 can arise, following [6, 8], at the expense of positive correlations along the

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following causative chain: large M_0 of a source $\Rightarrow$ large geological age ("maturity") of the fault $\Rightarrow$ large thickness of the damaged zone $\Rightarrow$ high L_{is} $\Rightarrow$ low f_{c3}. It should be clarified that earthquake sources both with large and with small M_0 can occur on long "mature" faults; whereas sources of large length and M_0 are improbable to happen on freshly formed, usually short faults. In such a situation it is reasonable to expect certain, but not sharply expressed, dependence f_{c3} on M_0.

The hypothesis [3, 4] regarding existence of L_{is} and f_{c3}, and their interrelation are not generally accepted. Their recognition is braked by the disputable character of the nature of the HF-cutoff of an observed spectrum of acceleration *record*. This cutoff arises through the joint action of the two different factors: (1) the HF cutoff of *source* spectrum at $f = f_{c3}$, as discussed above; and (2) *frequency-dependent losses* of energy of waves during their *propagation* to the receiver. It is not easy to split these effects; therefore the observed HF spectral cutoff is often treated as the pure effect of losses. When in [1, 9] the named effects where reliably separated, the sufficient amount of data was obtained for the comparative analysis of behavior of frequencies f_{c1}, f_{c2}, and f_{c3}.

Within the frames of the model [10], an earthquake source is formed through propagation, over its area, of a slipping zone or "slip pulse"; its width $L_r \approx v_r T_r$. This view is accepted further. The analysis is carried out on the basis of the simple idea: if L_{is} exists and it is related to T_{is}, and $T_{is} = T_{c3} = 1/f_{c3}$, it is possible, using f_{c3}, to normalize f_{c1} and f_{c2}, and thus to obtain dimensionless source parameters; then their relationship can be examined. In such a manner, the normalized rupture duration $\tau_1 = T_{c1}/T_{c3} = f_{c3}/f_{c1}$, and the normalized local rise time $\tau_2 = T_{c2}/T_{c3} = f_{c3}/f_{c2}$ are introduced. It is useful to introduce also the normalized size of the source λ_1. In this case it is possible to set normalized version of rupture velocity v_r to unity; then $\lambda_1 = \tau_1$; the dimensionless width of a slip zone $\lambda_2 = \tau_2$ is introduced similarly. Let is introduce the notion of an elementary slipping area on a fault, with its size equal to unity. Then the growth of an earthquake source can be imagined as process of consolidation of a finite number of elementary areas that join and form an aggregate or tight cluster. As an independent variable of scaling, the linear size of the source $\lambda_1 = \tau_1$ is further used, and the dependent variable τ_2 is a number of elementary areas across the "instant" width of growing front.

Growth of surfaces by aggregation of elementary objects and the increase of roughness of boundaries (or growth fronts) that accompany such a growth was intensively studied last decades in such broad contexts as: the advancement of a front of sedimentation of material in the form of suspended particles or atoms; the movement of the front of etching of material; the movement of front of wetting through a porous substrate, the propagation of a front of forest fire; the growth of cracks, and so forth [11, etc.]. The scale of formed roughness was described through root mean square width w of the random front of a growing area. Increase of w in time t usually follows power law $w(t) \sim t^{\beta_g}$, where β_g is «growth exponent». The comparison of observed characteristics of scaling $\tau_2(\tau_1)$ with the results of the mentioned studies can give the new information on laws of formation of earthquake sources. Using this approach, it was found out that the observed relationships $\tau_2(\tau_1)$ are quite comparable to typical laws of random growth.

Studying scaling within earthquake populations is an important source of information on the structure of an earthquake source. In such a study, one may employ an important, known, but rarely formulated explicitly concept of "empirical self-similarity", when one considers an already formed source of some earthquake as an approximate analogue of a certain stage in the formation of a source of a larger earthquake. It is conceptually close to the so named "cascade model" of source formation. Self-similarity is a known property of the crack model of an earthquake source [12]. If the idea of self-similarity is true, the initial part of a record from a large source should be similar to a record from a small source; observations generally support this idea.

Usually, the studies of scaling of sources deal with such rupture parameters as M_0, the length L, the area, the rupture time T, seismic energy E, and also spectral parameters. From seismograms one can restore source spectrum. This spectrum (Fig. 1) consists of a number of branches with behavior of the f^{γ} kind. The position of the crossover or corner point at the interface of two branches gives the key information in the form of corner frequency. The lowest of these, $f_{c1} = 1/T_{c1}$ (the bend from f^0 to $\approx f^{-1}$) is close to $1/T$. For T, scaling is known rather well, this gives the indirect information regarding f_{c1}. As for scaling of f_{c2}, (the bend from $\approx f^{-1}$ to f^{-2}, comparable to $1/T_r$) there are some data, however in a limited amount. A great volume of the information is accumulated for the frequency f_{c0}, which is close to $(f_{c1} f_{c2})^{0.5}$; it is this parameter which is commonly called "corner frequency". To extract the information on the behavior of f_{c1} and f_{c2} from the trend of f_{c0} may be problematic. The third corner frequency, f_{c3} (the bend from f^{-2} to $f^{-(3\text{-}4)}$) is studied insufficiently, even the concept of its reality is not a standard one.

Recently [1, 9], for the study of scaling, source spectra for more than 500 earthquakes of Kamchatka have been analyzed in the frequency band 0.1–30 Hz using the digital data of seismic station PET. The range of magnitudes is 3.8–6.5, the range of distances is 80–220 km. To reduce S-wave spectra to the source, attenuation properties of medium around PET were first studied [9] and then taken into account. Then

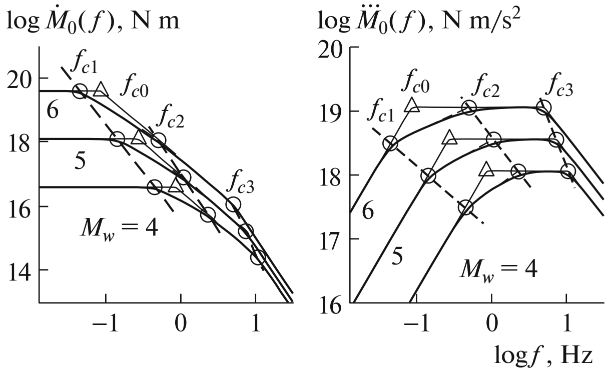

Fig. 1. The accepted schematic diagram of scaling of earthquake source spectra. Two variants are shown: common (displacement) source spectra $\dot{M}_0(f)$ (top), and acceleration source spectra $\ddot{M}_0(f)$ (bottom); these spectra are proportional to the spectra of displacement and acceleration of body waves.

corner frequencies f_{c1}, f_{c2}, and f_{c3} were determined (when possible), and their relationship with M_0 has been examined (Fig. 2). An observed trend of data for a corner frequency f_{ci} is described by slope parameter in log-log scale, $\alpha_i = \frac{-d\log f_{ci}}{d\log M_0}$. For f_{c1}, $\alpha_1 = 0.315 \pm 0.019$; such scaling approximately agrees with the usual hypothesis of similarity of the sources, that predicts $\alpha_1 = 1/3$; further $\alpha_2 = 0.176 \pm 0.017$ and $\alpha_3 = 0.103 \pm 0.025$. The scaling trend $f_{c2} \sim M_0^{-0.18} \sim f_{c1}^{\ 0.54}$ demonstrates expressed violation of similarity; the trend $f_{c3} \sim M_0^{-0.10} \sim f_{c1}^{\ 0.3}$ violates similarity even more dramatically. The empirical tendency $\alpha_1 > \alpha_2 > \alpha_3$ obviously bears important information on the properties of earthquake sources; to extract it is a challenge.

Following the program described in the prologue, for each spectrum, the normalized parameters $\tau_1 = f_{c3}/f_{c1}$ and $\tau_2 = f_{c3}/f_{c2}$ have been calculated. The τ_1 value is the duration of the source in units of T_{is}. This unit varies from one source to another, and on the average is close to 0.1 s. Simultaneously, $\tau_1 = \lambda_1$ defines the size of the source in L_{is} units, whose value is on the order of 300 m. The relationship $\tau_2(\tau_1)$ was examined; this dependence shows wide scatter (Fig. 3). The fields B, H, D and R, outlined on Fig. 3, envelope points that represent earthquake sources of qualitatively different space-time structure. In area B the slipping zone occupies entire source, thus $f_{c2} \approx f_{c1}$ and the source spectrum manifests a single bend from f^{-0}

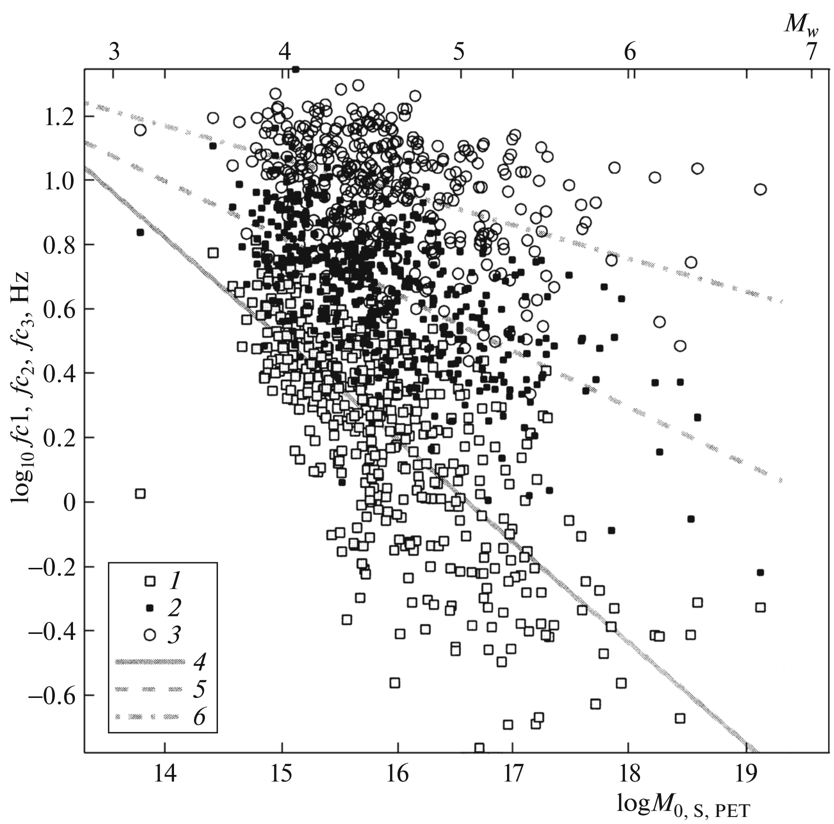

Fig. 2. Values f_{c1} (*1*), f_{c2} (*2*) and f_{c3} (*3*), picked from attenuation-corrected observed S-wave spectra of PET, and corresponding lines of orthogonal regression vs. log M_0 (*4, 5,* and *6*).

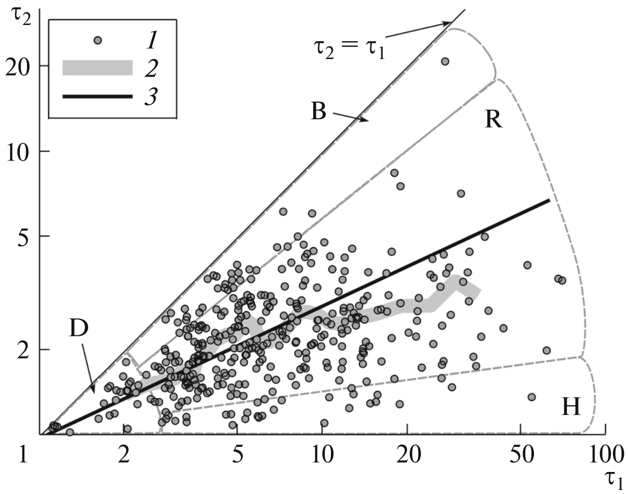

Fig. 3. Dimensionless local rise time τ_2 as a function of dimensionless duration of rupture τ_1. (*1*) data for individual earthquakes; (*2*) the line of running median; (*3*) the line of orthogonal regression.

branch to f^{-2} branch; it is a classical Brune's spectrum [13] with $\varepsilon = 1$. On the contrary, in the field H the slipping zone is narrow ($L_r \ll L$), and this behavior is close to models [10, 14]. At $f_{c1} \approx f_{c2} \approx f_{c3}$ (area D) the source spectrum follows the classical model "ω^{-3}"; the rupture at first expands approximately following the Kostrov's model [12], and then it slows down and is smoothly braked following Dahlen [15]. The central area R is that of "regular" sources whose behavior vary around the average trend, seen as the line of medians.

The slope of the line of orthogonal regression $\beta = \dfrac{-d\log\tau_2}{d\log\tau_1}$ is $\beta = 0.47 \pm 0.043$. This observed average behavior $\tau_2(\tau_1)$ can be compared to values β_g for existing models of random growth. In particular, the known model of Eden, appropriate for our instance, predicts $\beta_g = 1/3$ for the case of growth that starts in unperturbed conditions from a point primer. However, when the boundary moves in the conditions of quenched disorder, the value of β_g can reach 0.65, sometimes more. The estimate obtained here is approximately in the middle of the interval of known model estimates, and it does not contradict the usual concept about appreciable heterogeneity of fault surface. At $\tau_1 > 10$ the average trend on Fig. 3 is slowed down somewhat, and the estimate of slope approaches $\beta = 0.35$. It is not clear whether this deviation from a linear trend is a real feature or it is a peculiarity of the studied data set.

Data seen on Fig. 3 are obtained by processing parameters f_{c1}, f_{c2}, and f_{c3} of individual earthquakes. It is also interesting to estimate β from average relationships of f_{c1}, f_{c2}, and f_{c3} vs. M_0; as it is easy to see, $\beta = (\alpha_2 - \alpha_3)/(\alpha_1 - \alpha_3)$. For conditions of Kamchatka (see Fig. 2) this results in $\beta = 0.34$. The real accuracy of this estimate is limited, and its difference with the estimate obtained from individual data was not considered essential.

It was also interesting to compare the revealed trends with similar trends for other regions; but only two cases with such data could be found. Aki [2] combined f_{c3} data for the Wesern USA and Sea of Japan to obtain a straight line with a slope $\alpha_3 = 0.106$. For the Western USA he also cites data that define $\alpha_1 = 0.344$ and $\alpha_2 = 0.235$. From these data one can obtain $\beta = 0.57$. T.G. Rautian (private communication) has supplied the author with data on f_{c1}, f_{c2}, f_{c3} and M_0 for many earthquakes of Central Asia for 1972–1990; estimates of these parameters were obtained from coda spectra. From this data set it is possible to estimate $\alpha_1 = 0.326$; $\alpha_2 = 0.180$, and $\alpha_3 = 0.071$; thus $\beta = 0.42$.

The range 0.34–0.57 obtained for observed estimates of β, despite their low accuracy, agrees with the expected range β_g = 0.33–0.65, that follows from known models of random growth of a boundary. In should be also noted that the developed approach predicts that straight lines of the trends $\log f_{c1}$ ($\log M_0$), log f_{c2} ($\log M_0$) and log f_{c3} ($\log M_0$) should converge approximately to a single point, because for a smallest earthquake f_{c1}, f_{c3} and, consequently, f_{c2} must coincide. Such a tendency can be clearly seen on Fig. 2. This behavior takes place also for each of two other studied data sets. These facts support the discussed concept.

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