ISSN 1028-334X, Doklady Earth Sciences, 2013, Vol. 448, Part 2, pp. 211–213. © Pleiades Publishing, Ltd., 2013.
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GEOPHYSICS

A Fractal Earthquake Source with a Slip Zone Generates Acceleration Time Histories with Flat Spectra[1]

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Presented by Academician E.I. Gordeev June 21, 2012

Received June 27, 2012

Abstract—An important problem of the studies of earthquake sources is to clarify the mechanism of formation of radiated source spectra of the ω^{-2} ("omega-square") kind, or equivalently, of flat acceleration spectra. This spectral model is well established empirically and has the status of a classical one in source seismology; however, it lacks adequate theoretical foundation. It is shown that spectra of the ω^{-2} kind can be explained by combining the following three concepts regarding source rupture: (1) the fault asperity model of Das-Kostrov; (2) the Andrews's concept that the field of the stress drop over the fault is a 2D flicker-noise with amplitude spectrum of the $1/k$ type; and (3) the hypothesis that the distance of propagation of Rayleigh waves from a failing spot on a fault is determined by the width of the slip zone associated with the rupture front.

Keywords: earthquake source, source spectrum, rupture propagation, fractal, self-similar, accelerogram.

DOI: 10.1134/S1028334X13020049

In 1967 Aki [1] showed that body wave displacements from an earthquake source have high-frequency spectral asymptotics of the ω^{-2} kind. It was found [2] that this assumption allows one to simulate successfully ground accelerations in the epicentral zone. The mechanism that generates commonly observed spectra of the ω^{-2} kind remains a puzzle, solution of which is interesting and important for applications. In the present communication, a numerical kinematic source model is constructed that generates spectra of the ω^{-2} kind.

(1) Calculations are carried out on the basis of the theory of fault asperity after Das and Kostrov [3, 4]. In [3] they considered an infinite fault with zero friction with an asperity, i.e., a welded patch of limited-size, loaded with shear. During the failure of the asperity, the rupture front passes across its area. This propagating front generates surface waves that run along surfaces of the fault, and also body waves P and S. It is believed that stress on the fault is released instantly after arrival of the rupture front. Velocity $\dot{u}^{SH,\infty}(\mathbf{x},t)$ in the SH wave in the far-field receiver at a point $\mathbf{x} = \{x, y, z\}$ can be written as

$$\dot{u}^{SH,\infty}(\mathbf{x},t) = A\int_{\Sigma}\tau(\xi)\delta\left(t-\left(\frac{R-\xi\cdot\gamma+t_{fr}(\xi)}{c}\right)\right)dS, \quad (1)$$

where $u^{SH,\infty}(\mathbf{x},t)$ is displacement, $\xi = \{\xi_1, \xi_2, 0\}$ is a point of the source, with its hypocenter at $\xi = 0$, $R = |\mathbf{x}|$; $\gamma = \dfrac{\mathbf{x}}{R}$; $c = c_S$ is S-wave velocity; Σ is the source asperity patch, with the characteristic size $2R_c \ll R$ and an element dS; $\tau(\xi)$ is the stress drop on dS; $\delta(\cdot)$ is the delta-function; $t_{fr}(\xi)$ is the arrival time of the rupture front at ξ; and the A factor combines constant coefficients, geometric spreading, and the radiation pattern of SH waves for the unit force. It is supposed that $\tau(\xi) > 0$; thus, (1) is a unipolar pulse. Qualitatively, $u^{SH,\infty}(\mathbf{x},t)$ is step function ($\sim H(t)$), smoothed by a window with duration $T_c \approx \dfrac{2R_c}{v_r}$, where v_r is the mean rupture front velocity; $v_r \approx c_S$. The amplitude of this step is

$$u^{SH,\infty}(\mathbf{x},\infty) = \int\dot{u}^{SH,\infty}(\mathbf{x},t)dt = A\int_{\Sigma}\tau(\xi)dS \equiv AF_0, \quad (2)$$

where F_0 is "the seismic force of a source." For the average amplitude of the signal (1), one obtains

$$\dot{u}^{SH,\infty}(\mathbf{x},t) \approx \frac{AF_0}{T_c}. \quad (3)$$

Similar formulas exist for P and SV waves. On this basis in [5, 6], the earthquake source description was proposed as a set of strong small spots or asperities; a similar model is put forward in [7]. Here the viewpoint is different: the entire earthquake source is considered as one big asperity.

The model in [3] has been further developed [4] for a fault with a frictionless region of finite size $2R_r$. In

[1] This article was translated by the author.

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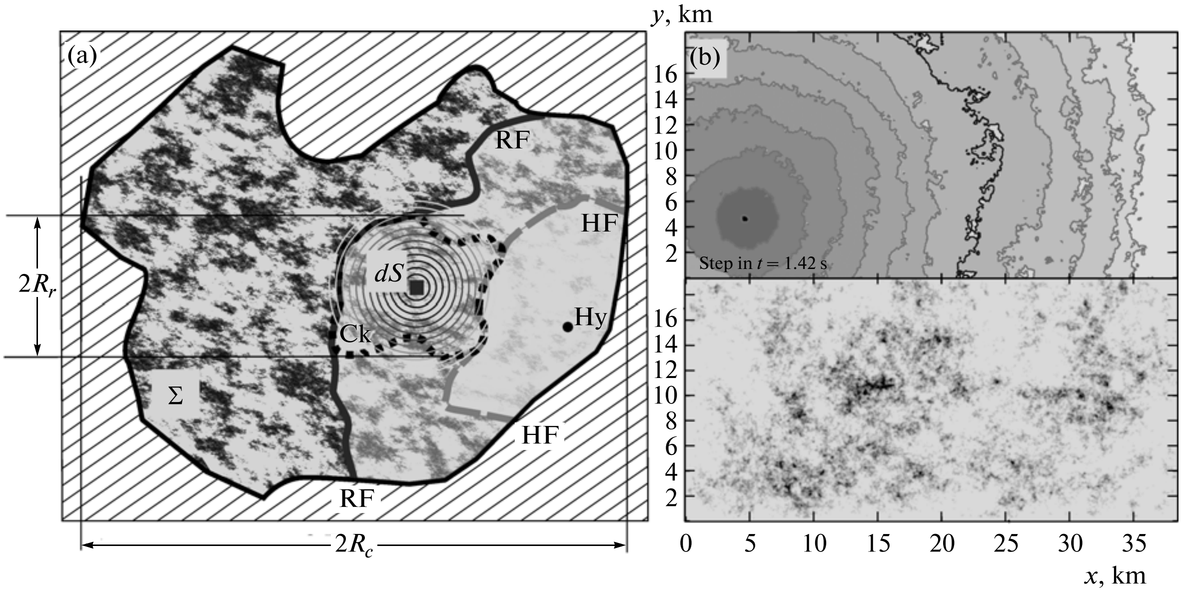

Fig. 1. (a) A cartoon depicting an earthquake source on an area Σ of the size $2R_c$. H—the hypocenter, RF—rupture front, HF—healing front. Ck is a patch on the fault of the size $2R_r$ (the dotted contour) where the propagation of Rayleigh waves radiated from dS is confined (a particular patch of this kind is associated with each particular dS). The rupture front and healing front are not strictly defined entities; they are understood as boundaries of the strip where the actual fractal front is localized. (b) An example of fields $t_{fr}(\xi_1, \xi_2)$ and $\tau(\xi_1, \xi_2)$ on a specimen of the simulated source. Wiggling isolines depict the positions of the fractal front each 1.42 s. Shades of gray code time: the later, the lighter. A particular example position of the front is emphasized by a bold line. Below an isotropic random field $\tau(\xi_1, \xi_2)$ with spectrum $\sim 1/k$ is depicted; shades of gray reflect amplitude; maxima are darker.

this case Rayleigh waves do not run to infinity, but die away on the boundary of this region, being converted into body waves. As a result, an additional term for the signal (1) arises which is followed by the delay of $\sim T_r = \dfrac{R_r}{c_R} \approx \dfrac{R_r}{c_S}$. As $t \to \infty$, this term accurately compensates for the displacement step $u^{SH,\infty}(\mathbf{x},\infty)$. Consequently the displacement signal obtains the habitual shape of a unipolar pulse:

$$u^{SH}(\mathbf{x},t) =$$
$$A\int_\Sigma \tau(\xi) G\left(t - \left(R - \xi\cdot\gamma + \frac{t_{fr}(\xi))}{c}\right)\right) dS \approx A\left(\frac{F_0}{T_c}\right)T_r, \quad (4)$$

where $G(\cdot)$ is a signal from an elementary radiator, appearing as an unipolar pulse of duration $\sim(1–2)T_r$, with an instant leading edge and stretched trailing edge,

$$G(t) = H(t) - \int \Lambda(s) H(t-s) ds. \quad (5)$$

Here $\Lambda(\cdot)$ is a window function with the integral equal to unity and duration $\sim T_r$. It is assumed that $G(\cdot)$ is the same for all, and taken as [7]:

$$G(t) = \begin{cases} H(t) - 0.5\left(1 + \dfrac{\cos \pi t}{T_r}\right), & t < T_r, \\ 0, \quad t > T_r. \end{cases} \quad (6)$$

with its amplitude $\sim \dfrac{AF_0T_r}{T_c}$ and duration $\sim T_c$ has its integral close to $\sim AF_0T_r = \dfrac{A}{c_S}F_0R_r$. As usual, it is related to the seismic moment M_0 of the source. This parameter is close to $M_0 = F_0R_r$, in agreement with [4].

In [4] the case is considered when Rayleigh waves propagate outside from an asperity ($R_r \gg R_c$). The opposite case $R_r \ll R_c$ is considered below. For correctness of such an approach, Rayleigh waves should be able to propagate (i.e., fault walls should be not welded) up to a certain distance R_r from the radiator dS located at the current point ξ of the rupture. When the rupture front runs through the vicinity of the point ξ, the mentioned condition indeed takes place, however only for a limited time T_r of sliding ("rise time"); and $R_r \approx c_ST_r$. Observations suggest that $T_r \ll T_c$.

It is assumed in [8] that $\tau(\xi_1, \xi_2)$ is a self-similar stochastic function with a Fourier spectrum shape close to $\dfrac{1}{k^\beta}$ (fractal) with $\beta \approx 1$; this hypothesis has been supported by inversions of real sources, and it is accepted in the following. In simulations, the distribution law for $\tau(\xi_1, \xi_2)$ is assumed to be lognormal. The scale of dispersion of values of $\tau(\xi_1, \xi_2)$ is fixed by setting the coefficient of variation $CV_\tau = \dfrac{\mathrm{Var}(\tau(\xi_1, \xi_2))^{0.5}}{\mathrm{E}(\tau(\xi_1, \xi_2))}$.

It is usually assumed that the running rupture front is a smooth line. However, the geometrical complexity of fronts is a necessary condition for formation of commonly observed incoherence of high-frequency radiation. Following [9] it is assumed that the rupture front has fractal geometry (is "lacy") and fills a strip of

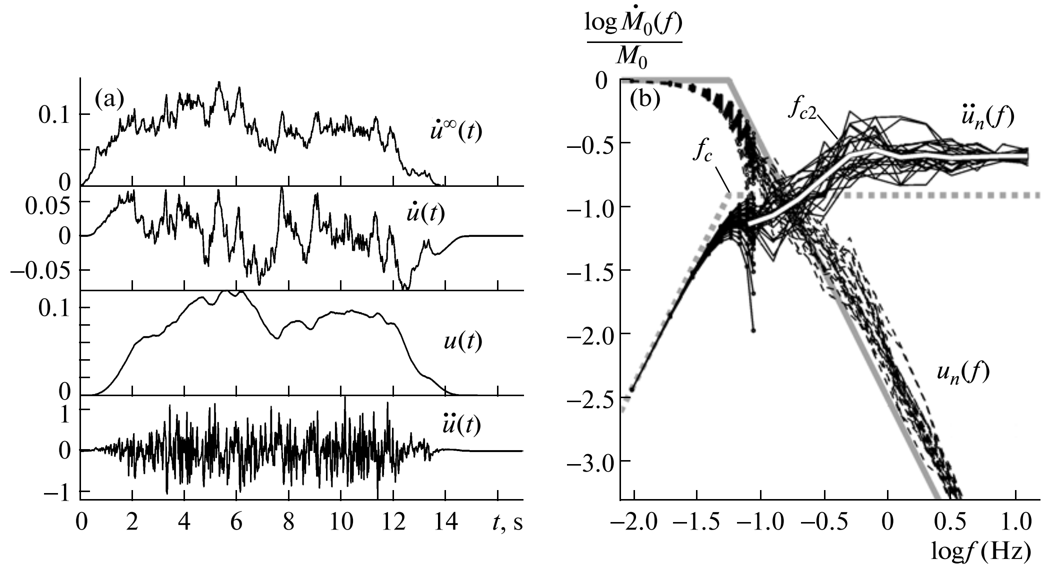

Fig. 2. Examples of simulation. (a) Signals $\dot{u}^{\infty}(t)$, $\dot{u}(t)$, $u(t)$, and $\ddot{u}(t)$ at the receiver. (b) Spectra $\ddot{u}_n(f)$ (solid) and $u_n(f)$ (dashes), initial and smoothed. The thick line is the geometrical average of $\ddot{u}_n(f)$ over 25 random tries. Gray lines are idealized spectra for the ω^{-2} model. The main corner frequency f_c is set as $\frac{1}{2\pi T_{rms}}$, where T_{rms}^2 is the second normalized central power moment for $u(t)$.

width $2R_r$. To achieve this, the moment of failure $t_{fr}(\xi_1, \xi_2)$ was set as

$$t_{fr}(\xi_1, \xi_2) = R(\xi_1, \xi_2) + S(\xi_1, \xi_2). \quad (7)$$

Here $R(\xi_1, \xi_2)$ is a random function that provides a fragmented structure and wiggling shape of the front; $R(\xi_1, \xi_2)$ is simulated as a self-similar function, with spectrum $\sim\frac{1}{k^{\delta}}$ and with uniform distribution law in the interval $[0, 2T_r]$. The term $S(\xi_1, \xi_2)$ provides regular behavior of the rupture at small k. It is accepted that $S(\xi_1, \xi_2) = \frac{|\xi - \xi_h|}{v_r}$, where v_r is a preset constant; ξ_h is the hypocenter (which provides the vertex of the cone $t = S(\xi_1, \xi_2)$).

(2) The procedure of numerical calculation includes the following (numerical values accepted in example calculations are given in parentheses):

—the choice of the size of the rectangular source (38 × 19 km), time step dt (0.025 s), distance step dx (0.075 km); setting v_r (3.0 km/s); c_S (3.5 km/s); setting ξ_h;

—setting parameters β (1.0), C_H (0.03), CV_τ (0.6), δ (1.3);

—generation of random fields $t_{fr}(\xi_1, \xi_2)$ and $\tau(\xi_1, \xi_2)$;

—calculating $u^{SH}(x,t) \equiv u(t)$ according to (1) for a ray along the normal to the fault (the analysis is confined by this case);

—calculating $u^{SH}(x,t) \equiv u(t)$ based on (2);

—determination of the normalized displacement spectrum $u_n(f)$ calculated from $u(t)$) through the Fourier transform, smoothed at high frequencies, and normalized by division by M_0; and also of the associated acceleration spectrum $\ddot{u}_n(f)$.

(3) Looking at Fig. 2a, one can perceive that signals $\dot{u}(t)$, $u(t)$, and $\ddot{u}(t)$ agree qualitatively with observations of real earthquake records. In Fig. 2b one can see that the smoothed acceleration spectrum is flat at high frequency, and that the second corner-frequency f_{c2} is present, a feature often noticed in the observed spectra. In general, a theoretically grounded and intrinsically consistent technique is created for simplified kinematic simulation of seismic waves radiated by an earthquake source in a broad frequency band.

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