Volc. Seis., 1992, Vol. 12(6), pp.723–733
Reprints available directly from the publisher

Printed in the United Kingdom

RELATIONSHIPS BETWEEN MAGNITUDE SCALES FOR GLOBAL AND KAMCHATKAN EARTHQUAKES

A. A. GUSEV and V. N. MELNIKOVA
Institute of Volcanology, Far East Division, USSR Academy of Sciences, Petropavlovsk-Kamchatskiy

(Received February 8, 1990)

Global relations between all generally used magnitude scales and moment magnitude M_W were constructed on a uniform basis. Similar scaling relations were derived for Kamchatkan earthquakes.

INTRODUCTION

Some practical problems of seismology require conversion from one magnitude scale to another. It was found long ago that average relations between magnitudes were often nonlinear. An example is a book published as far back as 1974 [6]. Utsu [29] carried out a systematic comparative study of magnitude scales assuming a nonlinear relationship between them and using the magnitude M_W, based on the seismic moment M_0. A similar approach was used in [2] where nonlinear magnitude relations with M_0 as the basis were utilized to construct spectral scaling relations. In recent years the routine determinations of seismic moment M_0 by the Harvard team headed by Dziewonski [13] and by the Sipkin team at NEIC, USA [28] provided an additional basis for updating magnitude relations, with the moment magnitude as the basis [17].

$$M_W = 2/3\, lg M_0 - 10.7 \qquad (1)$$

The M_W scale has the following advantages: (1) the distribution $N(M_W)$ (frequency-magnitude relation) is usually very close to being linear for large M, nonlinearity usually arising for the other magnitudes. This circumstance makes estimates of repeat times for great events more reliable; and (2) the scale has a well-defined physical meaning, hence extrapolations based on it are more likely to be successful. In many cases, however, two different magnitudes M_i and M_j need be related directly. A set of (M_i, M_W) relations is derived below and carefully tested from this point of view; it is shown that the error for the successive conversion $M_i \rightarrow M_W \rightarrow M_j$ is of the same order as the error for the direct regression ($M_i \rightarrow M_j$). This simplifies the conversion, because analysis of all pairs becomes superfluous. In this paper we used shallow earthquakes.

METHOD USED TO DETERMINE RELATIONSHIPS BETWEEN MAGNITUDE SCALES. NOTATION

In this study we used source data from several catalogs: the ESSN catalog (Unified Seismic Observation System), USSR; the Kamchatka regional catalog; NEIC, USA; the Harvard team catalog; Abe's catalog [11]; and data from [25]. We also used some magnitude relations available in the literature. Estimation of linear relations requires methods like that of orthogonal regression. As standard methods of this type are not available for nonlinear relations, we have developed a simple graphical technique. We called it a method of a local orthogonal median. It consists of five steps: (1) find the center of gravity of a data point cluster and draw a straight line through it close to the line of orthogonal regression (LOR); (2) rotate the set of axes so that the LOR makes a new *x*-axis; (3) divide the cluster into several (4–8) groups by vertical lines; (4) find the medians of the groups and draw them as horizontal bars (a step function) between the vertical lines; and (5) draw a smooth line that fits the step function.

This procedure may produce large errors when one of two magnitudes is represented by censored data. This may be the case when the lower thresholds for magnitude determination are appreciably different, for instance, in the case of an M_S and m_b relation. (In that case the hypothetical points for all earthquakes are censored at $M_S \approx 5$, and the $m_b(M_S)$ function looks steeper around $M_S = 5$ than it really is). We did not succeed to find a reliable and simple method for eliminating the effect of censoring and had to give up plotting estimates near that edge of the cluster where the censoring was significant.

This approach is correct if the errors of the two magnitudes M_i and M_j are about equal. Otherwise, the scales along the axes need be modified so as to make them equal (see [30]). For example, it is convenient to plot energy class estimates on a scale contracted by a factor of two. It should be pointed out that the theory of orthogonal regression is no more than a useful recipe in our case. This theory assumes (see, for instance, [30]) that deviations from the regression line $y(x)$ are due to "internal" errors in x and y rather than to a real scatter between two accurately measured quantities, which is the case in seismological practice.

We use the following notation. We denote the surface wave magnitude M_S (measured on a horizontal or on a vertical component) as M_S^{GR} when the Gutenberg formula is used, as M_S^{US} when the "Prague" formula is used and the period T = 17–23 s (this is the NEIC approach), and as M_S^{OB} when the Prague formula, a maximum amplitude, and an appropriate period are used (this is the ESSN approach). The m_{PV} magnitude obtained with medium-period and long-period instruments is denoted m_B, the synonyms being m_{PV}^{SK} and $m_{PV}(B)$ MPLP. The short period m_{PV} magnitude is denoted m_b for m_{PV} NEIC with Benioff instruments and m^{SKM} for m_{PV} ESSN with SKM-3 instruments. We do not use $m_{PV}(A)$, MPSP for reasons that are explained below. M_L is the Richter local magnitude, M_{JMA} is the JMA magnitude, K^{R60} is Rautian's energy class [7], and K^{F68} is Fedotov's energy class [9].

AVERAGE GLOBAL M_W, M_S AND m_B RELATIONS

The $M_S^{GR}(M_W)$ relation for large magnitudes was derived from M_S data obtained by Abe [11] for 1916–1980 and from M_W data after [25]. Following Abe, we put $M_S^{US} \equiv M_S^{GR} + 0.18$ and used M_S^{US} alone for our analysis. Moderate and low magnitudes were taken from [12] and from the Harvard and NEIC catalogs.

The $M_S^{OB}(M_W)$ relation was determined from the Harvard and ESSN catalogs for small and moderate magnitudes and the hypothesis $M_S^{OB} = M_S^{US}$ was tested using data from 1968 on for large magnitudes. Difference between M_S^{OB} and M_S^{US} was found to be small (below 0.05) for M_S greater than 5. The results are presented in Figure 1.

The following relation holds for $M_S < 6$

$$\lg M_0 \approx M_S^{US} + 19.24 \tag{2}$$

The relation between m_B and M_W for large magnitudes was derived using Abe's data for m_B and data from [25] for M_W. Moderate and small magnitudes were taken from the ESSN catalog for m_B and from the Harvard data for M_W. The result is nearly identical with that derived by substituting the relation $M_S^{GR}(M_W) = M_S^{US}(M_W) - 0.18$ into the Abe-Kanamori formula (see [11])

$$m_B = 0.65 M_S^{GR} + 2.5 \tag{3}$$

The result is given in Figure 1. The hint at saturation at $M_W \approx 9$ may be spurious due to an absence of intraplate events with $M_W = 9$–9.5; intraplate events of $M_W = 8$–9 displaced the mean m_B upward.

RELATIONS BETWEEN m_b, m^{SKM} and M_W

The justification of our notation is as follows. As regards the short period m_{PV} scale, the magnitude value is known to depend on the pendulum period: the m_b and m^{SKM} scales diverge from one another even for low M_W where the respective techniques must in principle yield identical results. This is why we gave up the standard notation MPSP. When $M_W > 6$ an additional requirement imposed by the NEIC on

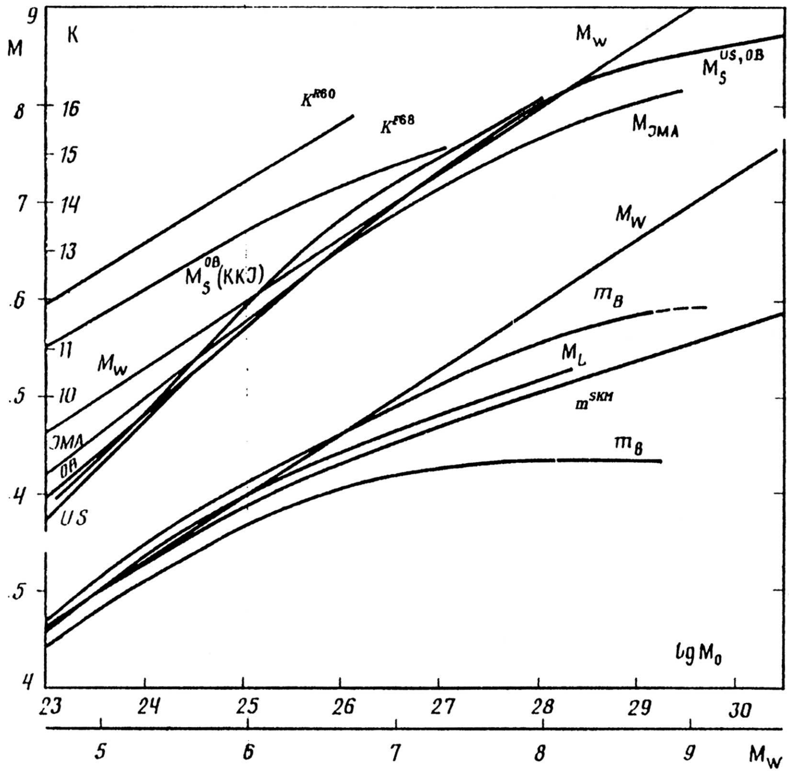

Figure 1 Average global magnitude relations plotted with M_0 and M_W as the arguments.

amplitude measurements becomes important: the greatest among the first few cycles should be selected (cf. [6]), so that m_b loses a precise meaning and is rapidly saturated at about 6.4. Koyama and Zheng [22] and Houston and Kanamori [18] have "corrected" the m_b value for many large earthquakes by reading the true maximum amplitude from seismograms and denoted the "corrected" m_b as m_b^* [22] and as $\hat{m}_b$ [18]. The m_b^* scale (which coincides with m_b when $m_b < 5.6$) turns out to be identical with m_{PV}^{SKM} apart from a constant term: $m_b^* \approx m^{SKM} - 0.18$. Figure 2 presents a summary plot of $m^{SKM}(M_0)$ and $m_b^*(M_0) + 0.18$. The plot is very well fitted by the straight line $m_{PV}^{SKM} = 0.525M_W + 2.86 = 0.35\ \lg M_0 - 2.75$ over a broad M_W range, $M_W =$ 6.6–9.5 (in agreement with [18]). One can notice an interesting asymmetry in the m^{SKM} deviations from the average curve, positive deviations greater than 0.4 being nearly absent.

The $m^{SKM}(M_W)$ plot from Figure 2 and the $m_b(M_W)$ plot borrowed from [12] are presented in Figure 1.

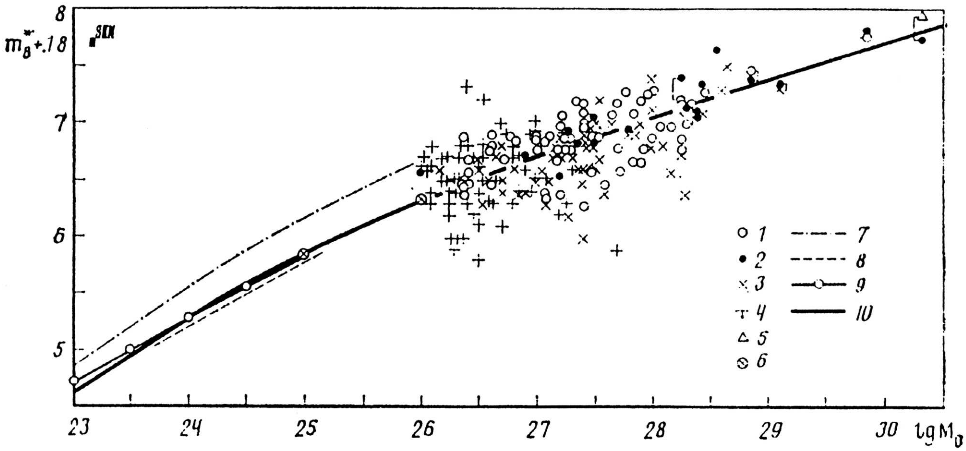

Figure 2 An m^{SKM} versus M_0 plot and m_b data corrected by +0.18: 1 - m_b^* after [22]; 2 - m_b after [18]; 3 - m^{SKM} after [25]; 4 - m^{SKM} after [15]; 5 - m_{PV} for a period of 1.5 s after [3]; 6 - centers of gravity for the Harvard data; 7 - trend for continental events; 8 - trend for interplate events; 9 - trend for Kamchatkan data with M_0 based on direct body waves; 10 - adopted relation. A bracket connects estimates for the same event.

REGIONAL SCALES

The M_L, K^{R60}, K^{F68}, M_{JMA} and other regional scales are important because they are used in areas of detailed seismological studies in the USA, the USSR, and Japan. The K scale with its various modifications is basically a magnitude scale; it is related to seismic energy by a regression relation [8]. Figure 1 presents the functions $M_L(M_W)$ [16], $M_{JMA}(M_W)$ based on the $M_{JMA}(M_W)$ and $M_{JMA}(M_S)$ plots from [29], $K^{R60}(M_W)$ from [7], and $K^{F68}(M_W)$ plotted by the writers.

One can see that $m^{SKM}(M_W)$, $M_L(M_W)$ and $0.5K^{F68}(M_W)$ have similar shapes. We remind that $M_L \sim \lg A$, while $m_{PV} \sim \lg(A/T)$ and $0.5K^{F68} \sim \lg(A/T)$. The division by the period does not involve any serious change in the relation, so that $0.5K^{F68}$ is on an average identical with M_L and m^{SKM} apart from a constant. This is certainly due to the use of short period ($T \approx 1$ s) instruments in all three scales.

All the above relations are presented in tabular form in Table 1.

MAGNITUDE RELATIONS FOR KAMCHATKA

This problem has been discussed in [1], [9]. Here we intended to improve the relations by using an extended database, comparing regional and global relations, and checking if there are systematic differences between individual data sets. We used the Kamchatkan, ESSN and NEIC catalogs and followed the technique described above.

Table 1 Magnitudes as functions of seismic moment: average global relations, relations for regional scales and average regional relations for Kamchatka (K) and Kamchatka-Kuril-Japan (KKJ).

Magnitude	lg M_0, dyne·cm							
	23	24	25	26	27	28	29	30
M_S^{GR}	3,58	4,58	5,54	6,34	7,12	7,82	8,23	8,45
M_S^{US}	3,76	4,76	5,72	6,52	7,30	8,00	8,41	8,63
M_S^{OB}	4,00	4,83	5,68	6,49	7,30	8,00	8,41	8,63
m_B	4,70	5,47	6,08	6,62	7,13	7,55	7,85	(7,98)
m^{CKM}	4,62	5,27	5,86	6,33	6,71	7,05	7,40	7,75
m_b	4,45	5,10	5,66	5,05	6,26	6,34	6,34	6,34
M_L	4,60	5,34	5,95	6,42	6,82	(7,16)	—	—
M_{JMA}	4,22	4,99	5,77	6,49	7,12	7,64	8,04	(8,27)
K^{F68}	11,08	12,22	13,36	14,37	(15,11)	(15,80)	—	—
M_S^{US} (KKJ)	3,73	4,68	5,65	6,47	7,25	(7,99)	—	—
M_S^{OB} (KKJ)	3,84	4,84	5,95	6,84	7,48	(8,04)	—	—
m_B (K)	4,98	5,62	6,23	6,77	(7,28)	—	—	—
m^{CKM} (K)	4,70	5,27	5,83	6,33	6,71	—	—	—
m_b (K)	4,46	5,06	5,63	5,99	6,23	—	—	—
M_W	4,63	5,30	5,97	6,63	7,30	7,97	8,63	9,30

Note. Less reliable values are given in parentheses.

Average regional relations were studied for M_{LH}, M_S, m^{SKM}, m_B, m_b and K^{F68}. The latter quantity is frequently subject to systematic error when $K^{F68} > 13$ (V. M. Zobin, private communication) owing to poor calibration of low-magnification instruments. For this reason instead of estimating K^{F68} from S waves as prescribed in [9] we used coda magnitudes [5] when dealing with $K^{F68} > 13$. The M_{LH} data for $M_{LH} <$ 4.5 were supplemented by estimates from records at the Petropavlovsk seismograph station corrected by +0.6 [6].

The $M_S^{OB}(M_W)$ and $M_S^{US}(M_0)$ relations were studied for the Kuril-Kamchatka region and Japan. We found some deviations (within 0.1) from the global relation for M_S^{US} and appreciable departures (as large as 0.33) for M_S^{OB}. As these regions showed little differences, one $M_S^{OB}(M_W)$ relation for both of them is shown in Figure 1 and Table 1; it is based on the Harvard [25] and ESSN catalogs.

The magnitudes m_B, m^{SKM}, m_b as functions of M_W for Kamchatka are almost identical with the global relations (m^{SKM}, m_b) or differ by a constant term: $m_b(M_W)$ is 0.15 above the global curve. The relations are presented in Table 1.

The standard deviations of our empirical relations with M_W as the argument are as follows: $\sigma(M_S^{OB}) = 0.35$; $\sigma(M_S^{US}) = 0.2$; $\sigma(m_B) = 0.25$; $\sigma(m_b) = \sigma(m^{SKM}) = 0.30$. For K^{F68} we have $\sigma = 0.65$.

The following linearized relations hold for the range $M_S^{OB} = 4–6$ ($K^{F68} = 11–14$):

$$K^{F68}=1.08M_S^{OB}+6.96, \quad (4)$$

$$m^{SKM}=0.57M_S^{OB}+2.47, \quad (5)$$

$$m_B=0.64M_S^{OB}+2.44. \quad (6)$$

The following relation is linear to a good approximation:

$$K^{F68}=2.00m^{SKM}+1.68\pm0.55. \quad (7)$$

The reason for this "integer" coefficient is not quite clear, since K^{F68} is based on appreciably higher-frequency data (1.3–4 Hz) than is m^{SKM} (commonly around 0.7 Hz).

Table 2 Mean deviations of magnitude relations for three data sets from the average regional relations.

Magnitude relation	*Data set*		
	A	*B*	*C*
$K^{F68}(M_S^{OB})$	0.20	-0.15	-0.80
$K^{F68}(M_S^{US})$	0.15	-0.12	-
$m^{SKM}(M_S^{OB})$	0.17	-0.09	-0.20
$m^{SKM}(M_S^{US})$	0.12	-0.03	-
Mean	0.10	-0.08	-0.30
N	60-100	30-40	30

The $K^{F68}(M_W)$ relation for small M_W is based on M_W data from the Harvard catalog and on the $K^{F68}(M_S^{US}(M_W))$ function where $M_S^{US}(M_W)$ is the average global relation. We also used our own M_0 estimates based on direct body waves.

We also examined the effects of depth and subregion on the short period magnitudes within the Kamchatka region. Here we used hypocenters deeper than 50 km. The best way to study the effect of the focal depth was to use $m^{SKM}(M_W)$, but we were unable to do so for lack of M_0 data. We found m^{SKM}, m_b and K^{F68} as a function of m_B. We assumed the shapes of the curves to be identical with the average regional ones and estimated mean deviations. For a depth range of 50–180 km we found $\Delta m^{SKM} = 0$, $\Delta m_b = 0.2$, and $\Delta K^{F68} = 0.4$ for samples ranging between 20 and 30. We failed to obtain reliable estimates for events deeper than 180 km.

The subregion effect was studied by the same procedure for events with H = 0–45 km using $K^{F68}(M_{LH})$, $K^{F68}(M_S)$, and $m^{SKM}(M_{LH})$. The following three data sets were treated separately: A – Petropavlovsk-Kamchatskiy area (52°–54°N, 158°–161°E), B – Ust-Kamchatsk area (55°–57°N, 163°–168°E), and C – the aftershocks of the Dec 15 1971 Ust-Kamchatsk earthquake. The results are presented in Table 2. The last but one line gives weighted means obtained for $\Delta m = 0.5\Delta K$. The sample size is indicated in the last line. We examined the K^{F68}–m_b, K^{F68}–m^{SKM}, and m_B–m^{SKM} relations by the same method and found deviations from the average regional trend to be nearly zero.

To conclude, this study revealed some real features in the source behavior at varying depths and in various subregions. In particular, difference between the B and C sets indicates an absence of a direct relationship between the source properties of a large earthquake and the properties of small and moderate magnitude events around it.

SOURCE PARAMETERS IN RELATION TO M_W AND M_0

It is useful to supplement the magnitude relations with a summary of relations for source parameters such as area S, length L, width W, mean slip D, and source function duration T. First, we give the relations assuming a rectangular source and the hypothesis of strict similarity

$$L \sim W \sim S^{1/2} \sim D \sim M_0^{1/3} \sim 10^{0.5M_W}, \quad T \sim L. \tag{8}$$

In other words, we assume that $w = L/W$ = constant, stress drop $\Delta\sigma = \mu CD/W$ = constant, and $v = L/T$ = constant. Here, C is a coefficient around one (0.5 to 3) which depends on the source geometry and distance to the surface [24]. Let $a = \Delta \lg L$ denote the correction for the deviation $\Delta\sigma$; averagely $a = 0$. We systematized data from [2], [10], [21], [25], [26] to determine the parameters involved in $L(M_W)$, $S(M_W)$,... The results presented below are for M_W = 5–9. We also attempted to construct a set of estimates based on observations that would be theoretically consistent. The most stable estimate for the source area is

$$\lg S\,[km^2] = M_W - 4.10 + 2a = 2/3 \lg M_0 - 14.80 + 2a. \tag{9}$$

For the typical value w = 2.5 this yields

$$\lg L\,[km] = 0.5 M_W - 1.85 + a = 1/3 \lg M_0 - 7.20 + a, \tag{10}$$

which is consistent with observations. The empirical estimate of D is

$$\lg D\,[cm] = 0.15 M_0 - 1.40 - 2a = 1/3 \lg m_0 - 6.75 - 2a. \tag{11}$$

This corresponds approximately to lg $\Delta\sigma$[bars] = 1.40+3a when C = 1. Using source times from [14], the T estimate for a full ("long-period") duration (v = 2.2 km/s) is

$$lg T[C] = lg L - 0.35 \tag{12}$$

and

$$lg T_{HF} = lg L - 0.55 \tag{13}$$

for a "short-period" duration (v = 3.5 km/s). The corner frequency can be estimated as

$$lg f_C = -lg T - 0.1. \tag{14}$$

The a value is influenced by focal mechanism and repeat time t, i.e., $a = a_M + a_R$, respectively. Following [4] and [10], we put $a_M = -0.1$ for crustal thrust events and +0.15 for strike slip earthquakes. Using repeat times from [20], a_R can be chosen as follows

t	=	< 70	70–300	300–2000	≥2000 years
a_R	=	0.15	0.05	−0.05	−0.15

The factor a_R in fact imitates the well-known interplate-intraplate earthquake factor [21] making its meaning more definite. Intraplate earthquakes show higher frequency energy for the same M_W, so m^{SKM} and m_b have to be corrected by +(0.3–0.4) (see Figure 2).

Following [20], we give a different set of relations for crustal events, assuming $W \sim L^{1/2}$

$$lg L[km] = 0.75 M_W - 3.60 + 2b \tag{15}$$

$$lg W[km] = 0.375 M_W - 1.45 + b \tag{16.1}$$

$$lg S[km^2] = 1.125 M_W - 5.05 + 3b \tag{16.2}$$

$$lg D[cm] = 0.375 M_W - 0.37 - 3b \tag{16.3}$$

$$lg \Delta\sigma\,[bars] = 1.40 - 4b, \tag{16.4}$$

where $b \approx 0.75a$.

Lastly, according to [27], strike slip events with abnormally large w > 6 obey the relations

$$lg D[cm] = lg L[km] + 0.1 \tag{17.1}$$

$$lgL^2W[km^3] = lgM_0 - 21.8 \qquad (17.2)$$

CONCLUSION

This summary of average global and Kamchatkan magnitude relations can be helpful for solving the following problems: conversion of different-type source data to a single scale, study of spatial and temporal spectral anomalies, comparison between regional estimates of strong motion parameters expressed in terms of regional scales to be used as empirical data for testing earthquake source models.

REFERENCES

1. A. V. Vikulin, *Volcanology and Seismology* No. 3 (1983)(cover-to-cover translation).
2. A. A. Gusev, *Volcanology and Seismology* No. 1 (1984)(cover-to-cover translation).
3. K. K. Zapolskiy and R. P. Solovieva, in: *Eksperimentalnaya seismologiya* (Experimental seismology)(Moscow: Nauka, 1971): 152-162(in Russian).
4. Yu. F. Kopnichev, *Korotkoperiodnye seismicheskie volnovye polya* (Short period seismic wave fields)(Moscow: Nauka, 1985)(in Russian).
5. V. K. Lemzikov and A. A. Gusev, *Volcanology and Seismology* No. 4 (1989)(cover-to-cover translation).
6. *Magnituda i energeticheskaya klassifikatsiya zemletryasenii* (Magnitude and energy classification of earthquakes). Vol. 1 and 2. (Moscow: IFZ AN SSSR, 1974)(in Russian).
7. Yu. V. Riznichenko, ed., *Metody detalnogo izucheniya seismichnosti* (Methods for detailed studies of seismicity). Trudy IFZ AN SSSR No. 9 (1960)(in Russian).
8. T. G. Rautian et al., *Eksperimentalnye issledovaniya seismicheskoi kody* (Experimental studies in the seismic coda)(Moscow: Nauka, 1981)(in Russian).
9. S. A. Fedotov, *Energeticheskaya klassifikatsiya kurilo-kamchatskikh zemletryasenii i problema magnitud* (Energy-based classification of Kuril-Kamchatka earthquakes and the magnitude problem)(Moscow: Nauka, 1972)(in Russian).
10. V. V. Shteinberg, *Izv. AN SSSR, Fizika Zemli* No. 7: 49-64(1983).
11. K. Abe, *Phys. Earth Planet. Inter.* **27**: 72-92(1981).
12. D. M. Boore, *Bull. Seismol. Soc. Amer.* **76**: 43-64(1986).
13. A. M. Dziewonski and J. H. Woodhouse, *J. Geophys. Res.* **88**: 3247-3271(1983).
14. M. Furumoto and I. Nakanishi, *J. Geophys. Res.***88**: 2191-2198(1983).
15. D. Giardini, A. M. Dziewonski, and J. H. Woodhouse, *Phys. Earth Planet. Inter.* **40**: 259-272(1985).
16. T. C. Hanks and D. M. Boore, *J. Geophys. Res.* **89**: 6229-6336(1984).
17. T. C. Hanks and H. Kanamori, *J. Geophys. Res.* **84**: 2348-2350(1979).
18. H. Houston and H. Kanamori, *Bull. Seismol. Soc. Amer.* **76**: 19-42(1986).
19. A. S. Jonson, *Seismol. Res. Lett.* **60**: 13(1989).
20. H. Kanamori and C. R. Allen, in: *Earthquake Source Mechanics, Geophys. Monograph* **37** (Washington D.C.: American Geophysical Union, 1986): 227-236.
21. H. Kanamori and D. L. Anderson, *Bull. Seismol. Soc. Amer.* **65**: 1073-1095(1975).
22. J. Koyama and S. H. Zheng, *Phys. Earth Planet. Inter.* **37**: 108-123(1985).
23. O. W. Nuttli, *Tectonophysics* **118**: 161-174(1985).
24. I. D. Parsons, J. F. Hall, and C. A. Lyzenga, *Bull. Seismol. Soc. Amer.* **78**: 931-945(1988).

25. G. Purcaru and H. Berckhemer, *Tectonophysics* **84**: 57-128(1982).
26. R. Sato, *J. Phys. Earth* **27**: 353-372(1979).
27. C. H. Scholz, *Bull. Seismol. Soc. Amer.* **72**: 1-14(1982)
28. S. A. Sipkin, *Bull. Seismol. Soc. Amer.* **76**: 1515-1541(1986).
29. T. Utsu, *Bull. Earthquake Res. Inst.* **57**: 465-497(1982)(in Japanese).
30. D. York, *Canadian J. Phys.* **44**: 1079-1086(1966).