RELATIONSHIPS BETWEEN MAGNITUDE SCALES FOR GLOBAL AND KAMCHATKAN EARTHQUAKES

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Global relations between all generally used magnitude scales and moment magnitude \( M_\mu \) were constructed on a uniform basis. Similar scaling relations were derived for Kamchatkan earthquakes.

INTRODUCTION

Some practical problems of seismology require conversion from one magnitude scale to another. It was found long ago that average relations between magnitudes were often nonlinear. An example is a book published as far back as 1974 [6]. Utsu [29] carried out a systematic comparative study of magnitude scales assuming a nonlinear relationship between them and using the magnitude \( M_\mu \) based on the seismic moment \( M_0 \). A similar approach was used in [2] where nonlinear magnitude relations with \( M_0 \) as the basis were utilized to construct spectral scaling relations. In recent years the routine determinations of seismic moment \( M_0 \) by the Harvard team headed by Dziewonski [13] and by the Sipkin team at NEIC, USA [28] provided an additional basis for updating magnitude relations, with the moment magnitude as the basis [17].

\[
M_\mu = 2/3 \lg M_0 - 10.7
\]  

(1)

The \( M_\mu \) scale has the following advantages: (1) the distribution \( N(M_\mu) \) (frequency-magnitude relation) is usually very close to being linear for large \( M \), nonlinearity usually arising for the other magnitudes. This circumstance makes estimates of repeat times for great events more reliable; and (2) the scale has a well-defined physical meaning, hence extrapolations based on it are more likely to be successful. In many cases, however, two different magnitudes \( M_i \) and \( M_j \) need be related directly. A set of \( (M_i, M_\mu) \) relations is derived below and carefully tested from this point of view; it is shown that the error for the successive conversion \( M_i \rightarrow M_\mu \rightarrow M_j \) is of the same order as the
error for the direct regression \((M_i-M_j)\). This simplifies the conversion, because analysis of all pairs becomes superfluous. In this paper we used shallow earthquakes.

**METHOD USED TO DETERMINE RELATIONSHIPS BETWEEN MAGNITUDE SCALES. NOTATION**

In this study we used source data from several catalogs: the ESSN catalog (Unified Seismic Observation System), USSR; the Kamchatka regional catalog; NEIC, USA; the Harvard team catalog; Abe's catalog [11]; and data from [25]. We also used some magnitude relations available in the literature. Estimation of linear relations requires methods like that of orthogonal regression. As standard methods of this type are not available for nonlinear relations, we have developed a simple graphical technique. We called it a method of a local orthogonal median. It consists of five steps: (1) find the center of gravity of a data point cluster and draw a straight line through it close to the line of orthogonal regression (LOR); (2) rotate the set of axes so that the LOR makes a new \(x\)-axis; (3) divide the cluster into several (4-8) groups by vertical lines; (4) find the medians of the groups and draw them as horizontal bars (a step function) between the vertical lines; and (5) draw a smooth line that fits the step function.

This procedure may produce large errors when one of two magnitudes is represented by censored data. This may be the case when the lower thresholds for magnitude determination are appreciably different, for instance, in the case of an \(M_S\) and \(m_b\) relation. (In that case the hypothetical points for all earthquakes are censored at \(M_S \approx 5\), and the \(m_b(M_S)\) function looks steeper around \(M_S = 5\) than it really is). We did not succeed to find a reliable and simple method for eliminating the effect of censoring and had to give up plotting estimates near that edge of the cluster where the censoring was significant.

This approach is correct if the errors of the two magnitudes \(M_i\) and \(M_j\) are about equal. Otherwise, the scales along the axes need be modified so as to make them equal (see [30]). For example, it is convenient to plot energy class estimates on a scale contracted by a factor of two. It should be pointed out that the theory of orthogonal regression is no more than a useful recipe in our case. This theory assumes (see, for instance, [30]) that deviations from the regression line \(y(x)\) are due to "internal" errors in \(x\) and \(y\) rather than to a real scatter between two accurately measured quantities, which is the case in seismological practice.

We use the following notation. We denote the surface wave magnitude \(M_S\) (measured on a horizontal or on a vertical component) as \(M_S^{GR}\) when the Gutenberg formula is used, as \(M_S^{US}\) when the "Prague" formula is used and the period \(T = 17-23\) s (this is the NEIC approach), and as \(M_S^{OB}\) when the Prague formula, a maximum ampli-
tude, and an appropriate period are used (this is the ESSN approach). The $m_{PV}$ magnitude obtained with medium-period and long-period instruments is denoted $m_b$, the synonyms being $m_{PV}^{SK}$ and $m_{PV}(B)$ MPLP.

The short period $m_{PV}$ magnitude is denoted $m_b$ for $m_{PV}$ NEIC with Benioff instruments and $m_{SKM}$ for $m_{PV}$ ESSN with SKM-3 instruments. We do not use $m_{PV}(A)$, MPSP for reasons that are explained below. $M_L$ is the Richter local magnitude, $M_{JMA}$ is the JMA magnitude, $K_{R60}$ is Rautian's energy class [7], and $K_{F68}$ is Fedotov's energy class [9].

**AVERAGE GLOBAL $M_W$, $M_S$ AND $m_B$ RELATIONS**

The $M_S^{GR}(M_W)$ relation for large magnitudes was derived from $M_S$ data obtained by Abe [11] for 1916-1980 and from $M_W$ data after [25]. Following Abe, we put $M_S^{US} = M_S^{GR} + 0.18$ and used $M_S^{US}$ alone for our analysis. Moderate and low magnitudes were taken from [12] and from the Harvard and NEIC catalogs.

The $M_S^{OB}(M_W)$ relation was determined from the Harvard and ESSN catalogs for small and moderate magnitudes and the hypothesis $M_S^{OB} = M_S^{US}$ was tested using data from 1968 on for large magnitudes. Difference between $M_S^{OB}$ and $M_S^{US}$ was found to be small (below 0.05) for $M_S$ greater than 5. The results are presented in Figure 1.

The following relation holds for $M_S < 6$

$$\log M_0 = M_S^{US} + 19.24$$

(2)

The relation between $m_B$ and $M_W$ for large magnitudes was derived using Abe's data for $m_B$ and data from [25] for $M_W$. Moderate and small magnitudes were taken from the ESSN catalog for $m_B$ and from the Harvard data for $M_W$. The result is nearly identical with that derived by substituting the relation $M_S^{GR}(M_W) = M_S^{US}(M_W) - 0.18$ into the Abe-Kanamori formula (see [11])

$$m_B = 0.65 M_S^{GR} + 2.5$$

(3)

The result is given in Figure 1. The hint at saturation at $M_W \approx 9$ may be spurious due to an absence of intraplate events with $M_W = 9-9.5$; intraplate events of $M_W = 8-9$ displaced the mean $m_B$ upward.

**RELATIONS BETWEEN $m_B$, $m_{SKM}$ AND $M_W$**

The justification of our notation is as follows. As regards the short period $m_{PV}$ scale, the magnitude value is known to depend on the pendulum period: the $m_b$ and $m_{SKM}$ scales diverge from one another even for low $M_W$ where the respective techniques must in principle yield identical results. This is why we gave up the standard notation MPSP. When $M_W > 6$ an additional requirement imposed by the NEIC on
amplitude measurements becomes important: the greatest among the first few cycles should be selected (cf. [6]), so that $m_b$ loses a precise meaning and is rapidly saturated at about 6.4. Koyama and Zheng [22] and Houston and Kanamori [18] have "corrected" the $m_b$ value for many large earthquakes by reading the true maximum amplitude from seismograms and denoted the "corrected" $m_b$ as $m_b^*$ [22] and as $\hat{m}_b$ [18]. The $m_b^*$ scale (which coincides with $m_b$ when $m_b < 5.6$) turns out to be identical with $m_{PV}^{SKM}$ apart from a constant term: $m_b^* \approx m_{PV}^{SKM} - 0.18$. Figure 2 presents a summary plot of $m_{PV}^{SKM}(M_0)$ and $m_b(M_0) + 0.18$. The plot is very well fitted by the straight line $m_{PV}^{SKM} = 0.525M_\nu + 2.86 = 0.35 \lg M_0 - 2.75$ over a broad $M_\nu$ range, $M_\nu = 6.6-9.5$ (in agreement with [18]). One can notice an interesting asymmetry in the $m_{PV}^{SKM}$ deviations from the average curve, positive deviations greater than 0.4 being nearly absent.

The $m_{PV}^{SKM}(M_\nu)$ plot from Figure 2 and the $m_b(M_\nu)$ plot borrowed from [12] are presented in Figure 1.

**Figure 1** Average global magnitude relations plotted with $K_0$ and $K_\nu$ as the arguments.
REGIONAL SCALES

The $M_L$, $K^{R60}$, $K^{F68}$, $M_{JMA}$ and other regional scales are important because they are used in areas of detailed seismological studies in the USA, the USSR, and Japan. The K scale with its various modifications is basically a magnitude scale; it is related to seismic energy by a regression relation [8]. Figure 1 presents the functions $M_L(M_N)$ [16], $M_{JMA}(M_N)$ based on the $M_{JMA}(M_N)$ and $M_{JMA}(M_S)$ plots from [29], $K^{R60}(M_N)$ from [7], and $K^{F68}(M_N)$ plotted by the writers.

One can see that $m^{SKM}(M_N)$, $M_L(M_N)$, and $0.5K^{F68}(M_N)$ have similar shapes. We remind that $M_L \sim \lg A$, while $m_{PV} \sim \lg(A/T)$ and $0.5K^{F68} \sim \lg(A/T)$. The division by the period does not involve any serious change in the relation, so that $0.5K^{F68}$ is on an average identical with $M_L$ and $m^{SKM}$ apart from a constant. This is certainly due to the use of short period ($T \approx 1$ s) instruments in all three scales.

All the above relations are presented in tabular form in Table 1.

MAGNITUDE RELATIONS FOR KAMCHATKA

This problem has been discussed in [1], [9]. Here we intended to improve the relations by using an extended database, comparing regional and global relations, and checking if there are systematic differences between individual data sets. We used the Kamchatkan, ESSN and NEIC catalogs and followed the technique described above.
Table 1 Magnitudes as functions of seismic moment: average global relations, relations for regional scales and average regional relations for Kamchatka (K) and Kamchatka-Kuril-Japan (KKJ).

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>23</th>
<th>24</th>
<th>25</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_S^{GR}$</td>
<td>3.58</td>
<td>4.58</td>
<td>5.54</td>
<td>6.34</td>
<td>7.12</td>
<td>7.82</td>
<td>8.23</td>
<td>8.45</td>
</tr>
<tr>
<td>$M_S^{US}$</td>
<td>3.76</td>
<td>4.76</td>
<td>5.72</td>
<td>6.52</td>
<td>7.30</td>
<td>8.00</td>
<td>8.41</td>
<td>8.63</td>
</tr>
<tr>
<td>$M_S^{OB}$</td>
<td>4.00</td>
<td>4.83</td>
<td>5.68</td>
<td>6.49</td>
<td>7.30</td>
<td>8.00</td>
<td>8.41</td>
<td>8.63</td>
</tr>
<tr>
<td>$m_B$</td>
<td>4.70</td>
<td>5.47</td>
<td>6.08</td>
<td>6.62</td>
<td>7.13</td>
<td>7.55</td>
<td>7.85</td>
<td>(7.98)</td>
</tr>
<tr>
<td>$m_{CKM}$</td>
<td>4.62</td>
<td>5.27</td>
<td>5.86</td>
<td>6.33</td>
<td>6.71</td>
<td>7.05</td>
<td>7.40</td>
<td>7.75</td>
</tr>
<tr>
<td>$m_b$</td>
<td>4.45</td>
<td>5.10</td>
<td>5.66</td>
<td>5.05</td>
<td>6.26</td>
<td>6.34</td>
<td>6.34</td>
<td>6.34</td>
</tr>
<tr>
<td>$M_L$</td>
<td>4.60</td>
<td>5.34</td>
<td>5.95</td>
<td>6.42</td>
<td>6.82</td>
<td>(7.16)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$M_{IM}$</td>
<td>4.22</td>
<td>4.99</td>
<td>5.77</td>
<td>6.49</td>
<td>7.12</td>
<td>7.64</td>
<td>8.04</td>
<td>(8.27)</td>
</tr>
<tr>
<td>$K_{FF68}$</td>
<td>11.08</td>
<td>12.22</td>
<td>13.36</td>
<td>14.37</td>
<td>(15.11)</td>
<td>(15.80)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$M_S^{US}(KKJ)$</td>
<td>3.73</td>
<td>4.08</td>
<td>5.65</td>
<td>6.47</td>
<td>7.25</td>
<td>(7.99)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$M_S^{OB}(KKJ)$</td>
<td>3.84</td>
<td>4.84</td>
<td>5.95</td>
<td>6.84</td>
<td>7.48</td>
<td>(8.04)</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$m_B(K)$</td>
<td>4.98</td>
<td>5.62</td>
<td>6.23</td>
<td>6.77</td>
<td>(7.28)</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$m_{CKM}(K)$</td>
<td>4.70</td>
<td>5.27</td>
<td>5.83</td>
<td>6.33</td>
<td>6.71</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$m_b(K)$</td>
<td>4.46</td>
<td>5.06</td>
<td>5.63</td>
<td>5.99</td>
<td>6.23</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$M_W$</td>
<td>4.63</td>
<td>5.30</td>
<td>5.97</td>
<td>6.63</td>
<td>7.30</td>
<td>7.97</td>
<td>8.63</td>
<td>9.30</td>
</tr>
</tbody>
</table>

Note. Less reliable values are given in parentheses.

Average regional relations were studied for $M_L^{SH}$, $M_S^1$, $m_{SKM}$, $m_B$, $m_b$ and $K_{FF68}$. The latter quantity is frequently subject to systematic error when $K_{FF68} > 13$ (V. M. Zobin, private communication) owing to poor calibration of low-magnification instruments. For this reason instead of estimating $K_{FF68}$ from S waves as prescribed in [9] we used coda magnitudes [5] when dealing with $K_{FF68} > 13$. The $M_L^{SH}$ data for $M_L^{SH} < 4.5$ were supplemented by estimates from records at the Petropavlovsk seismograph station corrected by +0.6 [6].

The $M_S^{OB}(M_W)$ and $M_S^{US}(M_0)$ relations were studied for the Kuril-Kamchatka region and Japan. We found some deviations (within 0.1) from the global relation for $M_S^{US}$ and appreciable departures (as large as 0.33) for $M_S^{OB}$. As these regions showed little differences, one $M_S^{OB}(M_W)$ relation for both of them is shown in Figure 1 and Table 1; it is based on the Harvard [25] and ESSN catalogs.

The magnitudes $m_B$, $m_{SKM}$, $m_b$ as functions of $M_W$ for Kamchatka are almost identical with the global relations ($m_{SKM}$, $m_b$) or differ by a constant term: $m_b(M_W)$ is 0.15 above the global curve. The relations are presented in Table 1.

The standard deviations of our empirical relations with $M_W$ as the argument are as follows: $\sigma(M_S^{OB}) = 0.35$; $\sigma(M_S^{US}) = 0.2$; $\sigma(m_B) = 0.25$; $\sigma(m_b) = \sigma(m_{SKM}) = 0.30$. For $K_{FF68}$ we have $\sigma = 0.65$.

The following linearized relations hold for the range $M_S^{OB} = 4-6$ ($K_{FF68} = 11-14$):
\[
K_{F68}^{OB} = 1.08M_S^{OB} + 6.96, \tag{4}
\]
\[
m_{SKM} = 0.57M_S^{OB} + 2.47, \tag{5}
\]
\[
m_B = 0.64M_S^{OB} + 2.44. \tag{6}
\]

The following relation is linear to a good approximation:
\[
K_{F68}^{OB} = 2.00m_{SKM} + 1.68 \pm 0.55. \tag{7}
\]

The reason for this "integer" coefficient is not quite clear, since \(K_{F68}^{OB}\) is based on appreciably higher-frequency data (1.3–4 Hz) than is \(m_{SKM}\) (commonly around 0.7 Hz).

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Mean deviations of magnitude relations for three data sets from the average regional relations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnitude relation</td>
<td>Data set</td>
</tr>
<tr>
<td>(K_{F68}^{OB}(M_S^{OB}))</td>
<td>(A)</td>
</tr>
<tr>
<td>(K_{F68}^{OB}(M_S^{US}))</td>
<td>0.20</td>
</tr>
<tr>
<td>(m_{SKM}(M_S^{OB}))</td>
<td>0.17</td>
</tr>
<tr>
<td>(m_{SKM}(M_S^{US}))</td>
<td>0.12</td>
</tr>
<tr>
<td>Mean</td>
<td>0.10</td>
</tr>
<tr>
<td>(N)</td>
<td>60-100</td>
</tr>
</tbody>
</table>

The \(K_{F68}^{OB}(M_H)\) relation for small \(M_H\) is based on \(M_H\) data from the Harvard catalog and on the \(K_{F68}^{OB}(M_S^{US}(M_H))\) function where \(M_S^{US}(M_H)\) is the average global relation. We also used our own \(M_0\) estimates based on direct body waves. We also examined the effects of depth and subregion on the short period magnitudes within the Kamchatka region. Here we used hypocenters deeper than 50 km. The best way to study the effect of the focal depth was to use \(m_{SKM}(M_H)\), but we were unable to do so for lack of \(M_0\) data. We found \(m_{SKM}\), \(m_B\) and \(K_{F68}\) as a function of \(M_H\). We assumed the shapes of the curves to be identical with the average regional ones and estimated mean deviations. For a depth range of 50–180 km we found \(\Delta m_{SKM} = 0\), \(\Delta m_B = 0.2\), and \(\Delta K_{F68} = 0.4\) for samples ranging between 20 and 30. We failed to obtain reliable estimates for
events deeper than 180 km.

The subregion effect was studied by the same procedure for events with $H = 0-45$ km using $K^{FM8}(M_{LH})$, $K^{FM8}(M_{S'})$, and $m^{SKM}(M_{LH})$. The following three data sets were treated separately: A - Petropavlovsk-Kamchatskiy area (52°-54°N, 158°-161°E), B - Ust-Kamchatsk area (55°-57°N, 163°-169°E), and C - the aftershocks of the Dec 15 1971 Ust-Kamchatsk earthquake. The results are presented in Table 2. The last but one line gives weighted means obtained for $\Delta m = 0.5\Delta K$. The sample size is indicated in the last line. We examined the $K^{FM8}-m$, $K^{FM8}-m^{SKM}$, and $m_{LH}-m^{SKM}$ relations by the same method and found deviations from the average regional trend to be nearly zero.

To conclude, this study revealed some real features in the source behavior at varying depths and in various subregions. In particular, difference between the B and C sets indicates an absence of a direct relationship between the source properties of a large earthquake and the properties of small and moderate magnitude events around it.

**SOURCE PARAMETERS IN RELATION TO $M_w$ AND $M_0$**

It is useful to supplement the magnitude relations with a summary of relations for source parameters such as area $S$, length $L$, width $W$, mean slip $D$, and source function duration $T$. First, we give the relations assuming a rectangular source and the hypothesis of strict similarity

$$L \sim W \sim S^{1/2} \sim D \sim M_0^{1/3} \sim 10^{0.5M_w}, \ T \sim L.$$  \hspace{1cm} (8)

In other words, we assume that $w = L/W = \text{constant}$, stress drop $\Delta \sigma = \mu CD/W = \text{constant}$, and $v = L/T = \text{constant}$. Here, $C$ is a coefficient around one (0.5 to 3) which depends on the source geometry and distance to the surface [24]. Let $a = \Delta l gL$ denote the correction for the deviation $\Delta \sigma$; averagely $a = 0$. We systematized data from [2], [10], [21], [25], [26] to determine the parameters involved in $L(M_w), S(M_w), ...$ The results presented below are for $M_w = 5-9$. We also attempted to construct a set of estimates based on observations that would be theoretically consistent. The most stable estimate for the source area is

$$l gS [km^2] = M_w - 4.10 + 2a = 2/3 l gM_0 - 14.80 + 2a.$$  \hspace{1cm} (9)

For the typical value $w = 2.5$ this yields

$$l gL [km] = 0.5M_w - 1.85 + a = 1/3 l gM_0 - 7.20 + a,$$  \hspace{1cm} (10)

which is consistent with observations. The empirical estimate of $D$ is

$$l gD [cm] = 0.15M_0 - 1.40 - 2a = 1/3 l gM_0 - 6.75 - 2a.$$  \hspace{1cm} (11)
This corresponds approximately to $\log \Delta \sigma \text{[bars]} = 1.40 + 3a$ when $C = 1$. Using source times from [14], the $T$ estimate for a full ("long-period") duration ($v = 2.2$ km/s) is

$$\log T[C] = \log L - 0.35$$  \hspace{1cm} (12)

and

$$\log T_{HFE} = \log L - 0.55$$  \hspace{1cm} (13)

for a "short-period" duration ($v = 3.5$ km/s). The corner frequency can be estimated as

$$\log f_c = -\log T - 0.1.$$  \hspace{1cm} (14)

The $a$ value is influenced by focal mechanism and repeat time $t$, i.e., $a = a_H + a_R$, respectively. Following [4] and [10], we put $a_H = -0.1$ for crustal thrust events and $+0.15$ for strike slip earthquakes. Using repeat times from [20], $a_R$ can be chosen as follows

$$t = \begin{array}{cccc}
    & < 70 & 70-300 & 300-2000 & \geq 2000 \\
    a_R = & 0.15 & 0.05 & -0.05 & -0.15
\end{array}$$

The factor $a_R$ in fact imitates the well-known interplate-intraplate earthquake factor [21] making its meaning more definite. Intraplate earthquakes show higher frequency energy for the same $M_H$, so $m_{SKM}$ and $m_p$ have to be corrected by $+(0.3-0.4)$ (see Figure 2).

Following [20], we give a different set of relations for crustal events, assuming $W \sim L^{1/2}$

$$\log L [\text{km}] = 0.75 M_H - 3.60 + 2b$$ \hspace{1cm} (15)

$$\log W [\text{km}] = 0.375 M_H - 1.45 + b$$  \hspace{1cm} (16.1)

$$\log S [\text{km}^2] = 1.125 M_H - 5.05 + 3b$$  \hspace{1cm} (16.2)

$$\log D [\text{cm}] = 0.375 M_H - 0.37 - 3b$$  \hspace{1cm} (16.3)

$$\log \Delta \sigma \text{[bars]} = 1.40 - 4b,$$  \hspace{1cm} (16.4)

where $b \approx 0.75a$.

Lastly, according to [27], strike slip events with abnormally large $W > 6$ obey the relations

$$\log D [\text{cm}] = \log L [\text{km}] + 0.1$$  \hspace{1cm} (17.1)
\[ \text{\textit{lgL}^2 W [km^3]} = \text{\textit{lgM}_0} - 21.8 \] (17.2)

**CONCLUSION**

This summary of average global and Kamchatkan magnitude relations can be helpful for solving the following problems: conversion of different-type source data to a single scale, study of spatial and temporal spectral anomalies, comparison between regional estimates of strong motion parameters expressed in terms of regional scales to be used as empirical data for testing earthquake source models.

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